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Essays in Corporate Finance

by

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Thesis

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Declarations

This thesis is submitted to the University of Warwick in support of my application for the degree of Doctor of Philosophy. It has been composed by myself and has not been submitted in any previous application for any degree.

The work presented (including data generated and data analysis) was carried out by the author except in the cases outlined below:

- Chapter 1 was done in collaboration with Dan Bernhardt;
- Chapter 2 was done in collaboration with John Thanassoulis.

Abstract

Chapter 1 shows how blockholder disclosure thresholds regulate market transparency and, in turn, hedge fund activism. We characterize how disclosure thresholds structure the complex interactions between (a) initial investors in a firm—who value the value-enhancing disciplining effects of activism on management, but incur costs trading with activists who know their own value-enhancing potential; (b) activists—who value higher thresholds when establishing equity stakes, but incur costs if high thresholds reduce real investment or discourage managerial misbehavior; and (c) firm managers—who weigh private benefits of value-reducing actions against potential punishment if activists intervene. When managerial behavior is sufficiently unresponsive to threats of activism, initial investors and society value tighter disclosure thresholds than activists whenever the costs of activism tend to be low, making the probability of activism insensitive to the level of activist trading profits. In contrast, activists value tighter thresholds when managerial behavior is responsive to potential activism.

In Chapter 2 we model a novel coordination problem between the shareholders of a company receiving a takeover offer. The willingness of a Board to defend itself from a takeover bid is reduced the greater the proportion of shareholders who sell-out early. Sophisticated shareholders therefore face a coordination problem; and their actions generate a novel feedback loop between the volume of shareholder sales and takeover outcomes. We use global games to derive and analyse the unique threshold- equilibrium. We show that rules which strengthen Boards' discretion to

make takeover judgments, or which weaken new shareholders' voting power, encourage shareholders to sell early; and that incentives to politically pressure Boards are greatest in jurisdictions with the greatest respect for shareholder rights.

Chapter 3 recognizes that firms' debt capacity affects their ability to compete in the product market, and the competitiveness of firms in the product market determines their ability to secure debt. I model the endogenous relation between product market competition and financial constraints by characterizing a trade credit transaction where a competitive retailer has incentives to not honour the debt extended by its supplier. With linear input prices, credit rationing arises endogenously in equilibrium if competitive pressure is strong enough. I show that a financially constrained retailer faces a lower price, and it can make higher profits due to its own financial constraints. With non-linear prices, the retailer might never be constrained, even though contractual frictions affect market outcomes.

Chapter 1

Blockholder disclosure thresholds and hedge fund activism

1.1 Introduction

Hedge fund activism mitigates agency problems that affect governance in publicly-traded companies with dispersed owners. An extensive empirical literature establishes that activist funds generate gains to their targets in terms of performance and stock prices (Brav et al. 2008; Clifford 2008; Klein and Zur 2009; Boyson and Mooradian 2011; Brav et al. 2015; Bebchuk et al. 2015). However, their financially-driven incentives (Brav et al. 2010; Burkart and Dasgupta 2015; Back et al. 2017) and the relative short-termism of their strategies (Brav et al. 2010) often generate controversy. In particular, activist hedge funds are, by nature, informed traders that profit from trading on their information advantages at the expense of uninformed shareholders. As a result, activists can impair real investment, destroying value (Leland 1992; Bernhardt et al. 1995).

Our paper models the inter-linkages between real investment, hedge fund

activism and managerial behavior, showing that hedge fund activism creates value when sufficiently limited, but that it can harm both uninformed investors and society otherwise. Our analysis contributes to the regulatory debate about the levels of optimal blockholder disclosure thresholds, when these thresholds limit trading profits of activist funds and hence their incentives to engage in costly, value-enhancing interventions. We determine how disclosure thresholds structure complex interactions between (a) initial investors in a firm—who value the direct and indirect value-enhancing disciplining effects of activism on management, but may incur costs trading with activists who are privately-informed of their value-enhancing potential; (b) activists—who value higher thresholds when establishing equity stakes, but incur costs if high thresholds deter real investment or discourage managerial misbehavior that is the source of their profits; and (c) firm managers—who weigh private benefits of malfeasance against potential punishment if activists intervene. We characterize how the optimal disclosure thresholds vary with economic primitives from the perspectives of uninformed investors, activists and a welfare-maximizing regulator. When managerial behavior is sufficiently unresponsive to threats of activism, initial investors and society value tighter disclosure thresholds than activists when the costs of activism tend to be low, so that the probability of activism is relatively insensitive to the level of activist trading profits. In contrast, when managerial behavior is responsive to potential activism, activists value tighter thresholds.

In 2011, senior partners at Wachtell, Lipton, Rosen & Katz (WLRK), a prominent law firm specializing in corporate and securities law and corporate governance, submitted a rulemaking petition—henceforth the WLRK (2011) Petition—to the Securities and Exchange Commission (SEC) advocating that rules governing the disclosure of blocks of stock in publicly traded companies be tightened.¹ WLRK argued that the US disclosure threshold of 5% allows activist investors to secretly

¹The WLRK (2011) Petition asks the SEC to update Schedule 13D reporting requirements to reduce a 10-day window between crossing the 5% threshold and the initial filing deadline, and to broaden definitions of beneficial ownership. External link to the WLRK (2011) Petition [here](#).

accumulate enough stock to control target companies. Empirical evidence shows that while activist funds create fundamental changes in targeted companies (Brav et al. 2008; Klein and Zur 2009), they typically own only about 6% of shares and only hold positions for short periods of time (Brav et al. 2010). This, WLRK argued, undermines the original purpose of Section 13(d) and damages market transparency and investor confidence. Academics responded, questioning the desirability of the proposed measures (Bebchuk and Jackson 2012; Bebchuk et al. 2013). They argued that a crucial incentive for activist funds is the ability to purchase stock at prices that do not yet reflect the value of their actions, and that tighter disclosure rules would discourage hedge fund activism. In turn, they argued that discouraging such activism would harm small investors, who would then not glean the value-enhancing benefits of hedge fund activism on corporate behavior.

Despite the importance of blockholder disclosure thresholds and the heated debate,² there has not yet been a comprehensive analysis to provide a rationale for this rule or guidance for potential adjustments. Why a threshold of 5%? When and how do interests of uninformed investors and activist hedge funds conflict? Can activists benefit from disclosure thresholds? Our paper sheds light on these issues. It presents a model of hedge fund activism and shows how disclosure thresholds affect (i) incentives of activist funds to engage in costly managerial disciplining; (ii) real investment of small uninformed investors; (iii) choices by managers of whether to pursue potentially value-destroying activities.

Activist funds profit from secretly acquiring undervalued stock and selling it

²The debate is built around interventions by leading academics and important figures in the industry. Examples of law experts promoting reductions in disclosure thresholds include the following interventions in the *Harvard Law School Forum on Corporate Governance and Financial Regulation*: “Section 13(d) Reporting Requirements Need Updating” and “13(d) Reporting Inadequacies in an Era of Speed and Innovation” by David A. Katz of WLRK in 2012 and 2015 respectively; “Activist Abuses Require SEC Action on Section 13(d) Reporting” and “Proposed Revisions to 13(d) Beneficial Ownership Reporting Rules” by Theodore N. Mirvis of WLRK in 2014 and 2016 respectively. Letters of both the Managed Funds Association and the Alternative Investment Management Association in (2013) to the Canadian Securities Administrators contain arguments by hedge funds against such proposals (external link [here](#)). Academic work against the WLRK Petition includes Bebchuk and Jackson (2012) and Bebchuk et al. (2013).

at higher prices after they intervene. Share prices typically rise sharply when an activist's presence is revealed because the market anticipates subsequent intervention, and [Bebchuk et al. \(2015\)](#) provide evidence that these post-disclosure spikes in share prices reflect the long-term value of intervention. Accordingly, the main source of rents for activist funds is the price change caused by their own interventions, and the value of the shares acquired prior to revealing themselves is key to their profitability (WLRK 2011 Petition, [Bebchuk and Jackson 2012](#)). A disclosure threshold limits the equity position that can be secretly acquired, reducing incentives to intervene. Importantly, the expected levels of activism affect the expected profitability of real investment by uninformed investors. In turn, this real investment affects the value of activist interventions, creating a feedback effect on the incentives of activists to participate. The optimal disclosure threshold policy for each party reflects the tensions faced with regard to the preferred level of market transparency.

Consider the tradeoffs faced by uninformed investors. Higher transparency (a lower disclosure threshold) reduces their trading losses, but it also reduces the willingness of hedge fund activists to intervene. In turn, this encourages management to pursue its own interests at the expense of shareholders. Uninformed investors value binding disclosure thresholds when the expected trading losses saved outweigh the disciplining benefits. They gain from the reduced shares that activists acquire when those shares are not needed to induce activism, but they are harmed when the share limit discourages activism. Their optimal disclosure threshold, when interior, trades these considerations off. In particular, uninformed investors value binding disclosure thresholds whenever the profit elasticity of activism is sufficiently small.

Despite the long-term value of hedge fund activism ([Brav et al. 2015](#); [Bebchuk et al. 2015](#)), researchers have found that activist funds tend to have short investment horizons,³ and that they acquire stock after targeting a firm.⁴ We model this by

³[Brav et al. \(2010\)](#) finds that the median duration of investment from when the Schedule 13D is filed until divestment is about nine months. [Brav et al. \(2008\)](#) and [Boyson and Mooradian \(2011\)](#) provide more details on the duration of activist hedge funds investment in target companies.

⁴[Bebchuk et al. \(2013\)](#) find that much of an activist's position is built on the day they cross the

considering a large informed (potential) activist fund that is external to the firm, and whose incentives to incur the cost of intervention are provided by the increase in the value of the stock that he acquires. That is, the activist’s sole source of rents is the increase in stock value due to intervention relative to the acquisition price, making activism directly related to block size.⁵

The activist endogenously determines how many shares to acquire. We develop a static dealership model (see e.g., [Kyle 1985](#)) in which the activist trades along with a random, uniformly-distributed measure of shareholders (initial investors) who receive liquidity shocks that force them to sell their shares. A competitive market maker sets price given the net order flow from the activist and initial shareholders. The activist’s order trades off between the benefits of a larger block size and the costs of the information revealed. This formulation is related in spirit to that in [Edmans \(2009\)](#), who introduces exponentially-distributed liquidity trade. This allows Edmans to solve for informed trade and expected profits in closed form. Here, uniformly-distributed liquidity trade serves the same purpose, delivering simple closed-form solutions. What matters for our analysis and findings are how an activist’s ex-ante expected trading profits are affected by disclosure thresholds at the moment the activist decides to intervene. In that sense, our qualitative insights extend naturally to a dynamic setting along the lines of [Back et al. \(2017\)](#), who focus on the dynamics of the stock acquisition process and the endogenous relation between block size and incentives for costly intervention.

The second key tension in our model is that the activist’s trading profits depend on the value of intervention, which is directly related to real investment—value-enhancing actions in larger companies have bigger impacts. When the expected losses of initial shareholders to the activist are too high relative to the benefits of

5% disclosure threshold.

⁵[Shleifer and Vishny \(1986\)](#) are the first to recognize the positive relation between monitoring incentives and block size, showing that large minority shareholders can alleviate free-riding in the disciplining of corporate management. [Edmans and Holderness \(2016\)](#) review the large literature that follows.

disciplining management, the ability to secretly acquire too many shares reduces real investment, reducing the profits that an activist can extract. The activist does not internalize the investment feedback effect in his trading because he participates only after initial investment has been sunk. A disclosure threshold can serve as a commitment device for an activist to limit his trade, and thereby raise real investment. Surprisingly, we establish the activist never wants a binding disclosure threshold just because it boosts real investment. That is, as long as managerial malfeasance is sufficiently insensitive to the threat of investor activism, we prove that this tension is always resolved against the investment feedback effect—the activist never wants to face a binding disclosure threshold.

The negative effect of market opacity on real investment captures the original concerns of the Williams Act (1968), which was designed to “alert investors in securities markets to potential changes in corporate control and to provide them with an opportunity to evaluate the effect of these potential changes”.⁶ Trading is a zero-sum game in which the activist’s expected trading profits represent expected trading losses to uninformed investors. When trading losses outweigh the benefits of monitoring, i.e., when the profit elasticity of activism is small, hedge fund activism harms uninformed investors, causing them to reduce investment. The opposite happens when the profit elasticity of activism is high. By regulating these trading transfers, disclosure thresholds affect real investment. This link between market efficiency and economic efficiency was first made in [Bernhardt et al. \(1995\)](#). Here, we identify twin real effects of informed trade by hedge fund activists: (i) it encourages activists to create value by intervening in underperforming companies, and (ii) it affects real investment.

The third strategic agent is the firm’s management. The manager can take

⁶Quote of the case resolution *Wellman v. Dickinson*, 682 F.2d 355, 365-66 (2d. Cir. 1982), citing *GAF Corp. v. Milstein*, 453 F.2d 709, 717 (2d Cir. 1971), cert. denied, 406 U.S. 910 (1972). (External link [here](#)). Used in “Section 13(d) Reporting Requirements Need Updating” by David A. Katz of WLRK in *Harvard Law School Forum on Corporate Governance and Financial Regulation*, 2012.

a value-destroying action to obtain private benefits, but she incurs a reputation cost if disciplined by the activist. Improvements in the performance and governance achieved by activists often come at the expense of managers and directors who see sharp reductions in compensation and a higher likelihood of being replaced (Brav et al. 2010; Fos and Tsoutsoura 2014). Keusch (2017) finds evidence that following activist campaigns, companies dismiss underperforming CEOs. As a consequence, the threat of being disciplined by an activist improves managerial performance (Gantchev et al. 2017). We capture this mechanism, recognizing the ex-ante disciplining role of hedge fund activists in discouraging managerial malfeasance. Since higher trading transfers make an activist more willing to act *if* management misbehaves, they also induce better behavior by management. We call this the managerial feedback effect.

The managerial feedback benefits uninformed investors, but, paradoxically, by reducing the likelihood that a manager pursues actions that benefit himself at the expense of shareholders, it reduces an activist’s opportunities to extract profit from its business of disciplining management. When managers are sensitive to the threat of activism, initial shareholders are happy to increase the block disclosure threshold, as their trading losses are only realized when the activist intervenes, so they are conditional on managers’ malfeasance. Raising the disclosure threshold both increases activists’ intervention rates (ex-post disciplining) and discourages malfeasance (ex-ante disciplining). The same mechanism represents a tension for an activist fund, which trades off higher conditional trading profits against a lower probability of profiting. When the activism elasticity of malfeasance is sufficiently large, i.e., when managerial feedback is strong, activist hedge funds benefit from tighter disclosure thresholds. Indeed, we establish that whenever activists value a binding disclosure threshold, it is always lower than that preferred by uninformed investors. In effect, the willingness of an activist hedge fund discourages excessively—from its perspective, but not shareholders—the desire of management to pursue its own interests

at the expense of shareholders. Shareholders gain from the activist’s willingness to engage without having to pay in terms of trading costs.

We also characterize the socially-optimal disclosure threshold and show that it rarely coincides with the preferred policies of uninformed investors or activists. Society (a regulator) does not internalize the transfer of trading profits from uninformed investors to the hedge fund, caring only about the expected value of the firm net of the cost of capital and the cost of activism. Intuitively, society is not directly concerned about trading in financial markets, but only the indirect real effects of such trading. We show that the socially-optimal disclosure threshold is always weakly between the thresholds preferred by shareholders and the activist hedge fund.

We next relate the paper to the literature. Section 2 studies a simple model of hedge fund activism in which managerial behavior is exogenous. Section 3 introduces blockholder disclosure thresholds and derives the optimal policies for the different parties. Section 4 endogenizes managerial behavior. Section 5 concludes. An Appendix contains all proofs.

1.1.1 Related Literature

This paper contributes to a rapidly growing body of research on hedge fund activism. The seminal work of [Shleifer and Vishny \(1986\)](#) introduced the role of blockholders as monitors of corporate management. More recently, research has focused on the relation between financial markets and the monitoring incentives of blockholders (see [Edmans and Holderness 2016](#) for a review).⁷ The literature on hedge fund activism, including our paper, reflects that disciplining management often *is* the business of blockholders. The key role of financial markets follows from the strategies of blockholders, which consist of acquiring stock in target companies before the price

⁷Broadly, this can be classified as “exit” ([Admati and Pfleiderer, 2009](#); [Edmans, 2009](#); [Edmans and Manso, 2011](#)); “voice” literature in shareholder interventions; and the interaction between the two ([Levit, 2017](#); [Back et al., 2017](#)).

reflects the value of their actions.

We motivate our main modelling assumptions using findings from the vast empirical literature on hedge fund activism. [Collin-Dufresne and Fos \(2015\)](#) and [Gantchev and Jotikasthira \(2017\)](#) provide evidence that hedge fund activists exploit liquidity sales to purchase stock in target companies. A host of papers document that activist funds enhance the value of these companies by disciplining management ([Brav et al., 2008](#); [Clifford, 2008](#); [Klein and Zur, 2009](#); [Boyson and Mooradian, 2011](#); [Brav et al., 2015](#); [Bebchuk et al., 2015](#)) through costly interventions ([Gantchev, 2013](#)). [Brav et al. \(2010\)](#), [Fos and Tsoutsoura \(2014\)](#) and [Keusch \(2017\)](#) provide empirical foundations for our assumption that managers in target companies are penalized when disciplined by activist funds. Our paper endogenizes firm value by assuming that investors react to the expected value of the company, which is determined by corporate governance. While this relation has not been established in the literature on hedge fund activism, [La Porta et al. \(2006\)](#) and [Djankov et al. \(2008\)](#) find evidence of higher investment in markets with more legal investor protection.

Some of our predictions have empirical support, while others remain to be tested. The model predicts that the stock price reaction that follows disclosure of an activist fund captures the value of their actions ([Bebchuk et al., 2015](#)), and that disclosure thresholds constrain their acquisitions ([Bebchuk et al., 2013](#)). [Gantchev et al. \(2017\)](#) provides evidence of the ex-ante disciplining effect of hedge fund activism.

Few papers have formally studied hedge fund activism. Most notably, our paper recognizes the role of financial markets on the incentives of activists to take positions in a target company and intervene. This property is shared with [Back et al. \(2017\)](#), who characterize the dynamic trading of an activist investor. As in our paper, they follow [Kyle \(1985\)](#) by introducing exogenous market uncertainty (liquidity trading) that provides camouflage for a blockholder's trades. [Back et al. \(2017\)](#) revisit the classic question of the relationship between liquidity and economic

efficiency, and show how the intervention cost function affects outcomes. Our paper simplifies the trading process (static) and the cost of intervention (fixed) in order to endogenize investment and study the role of market transparency, i.e., of blockholder disclosure thresholds. In this way, the two papers complement each other.

Market liquidity also plays a key role in our model. The activist fund is initially external to the target company, so liquidity camouflages its purchases of shares and diminishes adverse price impacts, making intervention more profitable. This positive relationship was first formalized by [Maug \(1998\)](#) and [Kahn and Winton \(1998\)](#) in the context of general blockholder interventions, and [Kyle and Vila \(1991\)](#) in the context of takeovers.⁸

Other analyses of hedge fund activism share with our paper the essential trade-off between the financial benefit of increasing a target company's value (and hence share price) and the cost of intervention. [Burkart and Lee \(2015\)](#) compare hedge fund activism with hostile takeovers in a complete information setting, and show that they can be regarded as polar approaches to the free-riding problem of [Grossman and Hart \(1980\)](#). [Burkart and Dasgupta \(2015\)](#) model hedge fund activism as a dual-layered agency model between investors, activists and managers. Activist funds compete for investor flows, and this affects their governance as blockholders. In their paper, funds inflate short-term performance by increasing payouts financed by leverage, which discourages value-creating interventions in economic downturns due to debt overhang. [Brav et al. \(2016\)](#) recognize the complementarity of costly interventions by distinct funds in the same target and model the resulting coordination problem.

Our paper is also related to the insider trading literature. In our model, the hedge fund activist is an informed trader that profits from trading with uninformed investors. This reduces the profitability of uninformed investors, who then reduce

⁸In contrast, [Coffee \(1991\)](#) and [Bhude \(1993\)](#) argue that liquidity facilitates exit when management underperforms, and therefore discourages intervention. In the dynamic analysis of [Back et al. \(2017\)](#), the relationship between liquidity and intervention depends on the structure of the cost function.

their investments. [Leland \(1992\)](#) and [Bernhardt et al. \(1995\)](#) first model this mechanism to study the welfare effects of insider trading. This literature focuses on the informational role of prices for investment and anticipation of future trading by uninformed agents with informed traders; our current paper combines this anticipation of future trading with how such informed trading provides incentives for managerial disciplining. A more direct link to the insider trading literature concerns the impact of mandatory disclosure rules for insiders (see [Huddart et al. 2001](#)).

Our paper is motivated by a regulatory debate that has been largely overlooked by the finance literature. Many calls for revisions of blockholder disclosure rules have been made by prominent lawyers, hedge funds and academics.⁹ [Bebchuk and Jackson \(2012\)](#) provide a comprehensive analysis in corporate law of the law and economics of blockholder disclosure thresholds, and [Bebchuk et al. \(2013\)](#) empirically analyse pre-disclosure accumulations of hedge fund activists. In line with their findings, our paper shows that lower disclosure can increase investor value. This is against a widely-held view that higher transparency *must* provide more investor protection,¹⁰ a view that ignores investor protection from activists.

1.2 Hedge Fund Activism

In this section we model hedge fund activism and characterize the inter-linkage with real investment. We consider a firm that raises capital for a project whose value depends on the initial investment by uninformed *investors* and a business plan that may be either good or bad. The *manager* can deliberately adopt the bad business plan in order to obtain private rents at the expense of shareholders. The bad plan reduces value for shareholders unless an outside *activist* hedge fund intervenes to discipline management and implement the good plan. All agents are risk neutral.

⁹ *Harvard Law School Forum on Corporate Governance and Financial Regulation* contains a discussion of interventions by different parties.

¹⁰See [Schouten and Siems \(2010\)](#) and references therein for the corporate law literature; and see [La Porta et al. \(2006\)](#) and [Djankov et al. \(2008\)](#) for papers in the economics literature that use ownership disclosure rules in an index of investor protection.

There are four dates, $t = 0, 1, 2, 3$. There is no discounting.

At $t = 0$ a continuous of dispersed investors invest capital k in a project, which is expected to pay off

$$V = f(k) [1 - \delta \cdot 1_{\{m=0\}}] \quad (1.1)$$

at $t = 3$. Here, f is a standard production technology with $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) \rightarrow \infty$. The indicator function accounts for the business plan $m \in \{0, 1\}$ implemented by the manager at $t = 1$. The good plan ($m = 1$) yields cash flows $f(k)$ to investors. The bad plan ($m = 0$) yields nothing with probability $\delta \in [0, 1]$. Equivalently, the bad plan destroys a proportion δ of the project's value. Investors are uninformed, unable to distinguish between the good and bad business plans. We initially assume that the manager adopts the bad business plan ($m = 0$) with exogenous probability z . Section 4 endogenizes managerial malfeasance. The marginal cost of capital is $r > 0$. Initial investors become shareholders that receive claims to terminal project payoffs that they may trade at date 2. We normalize the measure of shares outstanding to one.

At $t = 2$, some initial investors receive liquidity shocks and must sell their shares in a competitive dealership market. These investors sell l shares, where l is uniformly distributed on $[0, b]$. We let $x(l)$ denote the associated density, and observe that $b \in [0, 1]$ is a measure of market liquidity. Also possibly present in the market is an activist, who is an outsider to the firm. The activist identifies managerial malfeasance when it occurs with probability λ . The activist can discipline management by incurring a fixed cost c , forcing the firm to shift from the bad business plan to the good one. The activist privately observes this cost c . Other market agents share a common prior that c is distributed on $[0, C]$ according to a strictly positive and weakly decreasing density g and associated cumulative function G . The activist chooses how many shares $\alpha \in [0, 1]$ to acquire, which we term his

position. A competitive *market maker* observes the net order flow $\omega = \alpha - l$ from the activist and initial investors, but not its components, and sets a price that equals expected project payoffs given ω , i.e., the market maker breaks even in expectation as in Kyle (1985).

To ease presentation, we assume that (i) the activist cannot act as a mere inside trader, only intervening when $m = 0$; and (ii) the activist disciplines management whenever he takes a position in the firm; i.e., he does not “cut-and-run” by selling his shares before engaging with management. We show in Appendix B that neither of these assumptions qualitatively affects the results.¹¹

At $t = 3$, the project delivers cash flows $f(k)$ if the manager implemented the good plan or if the activist disciplines the manager. Otherwise, expected cash flows are $(1 - \delta)f(k)$. Figure 1.1 summarizes the sequence of events.

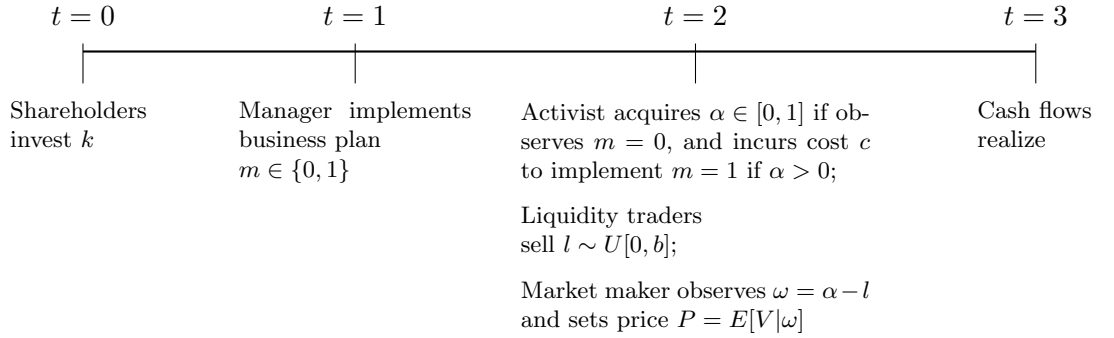


Figure 1.1: Time line

The parameters δ and z capture the severity of the agency problem between management and ownership. If $\delta = 0$, both business plans yield cash flows $f(k)$,

¹¹Appendix B.1 allows the activist to acquire stock when the business plan is good ($m = 1$) and there is no need for him to intervene. This *increases* his information rents without affecting the net value of the project. This hurts uninformed investors reducing their investment. Appendix B.2 introduces a new liquidity shock to allow the activist to sell shares after the price increase that follows disclosure, and shows that an equilibrium with “cut and run” does not exist unless binding disclosure thresholds are very low. In reality, while activists may “cut-and-run”, empirical evidence shows that hedge fund activism increases the value of target companies via costly disciplining (see Gantchev (2013) for the costs of activism, and Bebchuk et al. (2015) for evidence on the value of hedge fund activism). Cutting and running becomes even less attractive when it impairs the reputation of activist funds, which Johnson and Swem (2017) find to be important for their profitability.

so there are no frictions between investors and the manager, and thus no room for managerial disciplining; and if $z = 0$ the manager always implements the good business plan. In contrast, $\delta > 0$ and $z > 0$ imply that the manager may destroy shareholder value to obtain private benefits, creating a potential role for hedge fund activism. In Section 4 we endogenize the probability z that the manager implements the bad business plan to study the effects of a strategic manager.

We assume that the activist correctly identifies the good business plan and can discipline management at cost c ,¹² and that the activist buys shares in the target company at a single time where shareholders (investors) face liquidity shocks. In practice, these processes are dynamic,¹³ with uncertain costs and outcomes.¹⁴ We abstract from these mechanics to study the incentives provided by financial markets. What matters for our analysis are the *expectations* that the activist forms about these costs and outcomes at $t = 2$ when deciding *whether* to attempt to discipline management. The decision is based on the balance between *expected* financial benefits and engagement costs, and the likely dynamic price impacts of trading—and not the particular paths that can be realized given a decision to move forward.

The static trading setting preserves the fundamental property that there is an adverse price effect via trading that reveals information to the market (see, e.g., [Bebchuk and Jackson, 2012](#)). The continuum of random liquidity sales l allows us to endogenize the activist’s position α ; and the uniform distribution yields a simple closed-form solution for this position. These properties facilitate our analysis of blockholder disclosure thresholds. The continuum of liquidity shocks differen-

¹²In practice, the intervention cost c depends on factors such as an activist’s ability to coordinate with other shareholders and managerial entrenchment. See [Back et al. \(2017\)](#) for how different functional forms for intervention costs affect the incentives to intervene in a dynamic setting.

¹³The dynamics of stock acquisition have been studied in the insider trading literature (see [Collin-Dufresne and Fos 2016](#) and references therein). With regard to hedge fund activism, [Collin-Dufresne and Fos \(2015\)](#) find evidence that activist funds select times of higher liquidity when they trade, while [Back et al. \(2017\)](#) analyse the incentives of “exit” versus “voice” during the trading process.

¹⁴For an analysis of the engagement process see [Gantchev \(2013\)](#), who builds a sequential decision model to estimate the costs of proxy fights and other stages of shareholder activism. See [Becker et al. \(2013\)](#) for details on the costs of launching a proxy fight.

tiates our model from most corporate finance models that feature simple discrete (typically binary) levels of liquidity trade. Our trading environment is similar in spirit to [Edmans \(2009\)](#), who assumes exponentially distributed liquidity purchases to characterize the sales of an informed blockholder.

1.2.1 Market Equilibrium

We solve recursively for the perfect Bayesian Nash equilibrium of this model. At $t = 2$ real investment has been sunk by uninformed investors and is observable to all parties, the manager adopted the bad business plan with probability z , and the activist observes malfeasance with probability λ . Uninformed investors receive liquidity shocks and trade simultaneously with the activist in the dealership market with pricing by the risk neutral market maker. At $t = 0$ investors anticipate the subsequent events and invest capital.

Trading

Proposition 1 summarizes the Bayesian Nash equilibrium in the subgame at date 2. The activist participation and trade is optimal given the market maker's pricing function, and the market maker's pricing function earns it zero expected profits given the activist's decisions.

Proposition 1 *At $t = 2$ real investment k is sunk and observable to all parties. If the activist observes managerial malfeasance ($m = 0$), and the cost of activism is sufficiently small, $c \leq c_t^*$ where*

$$c_t^* = z [1 - \lambda G(c_t^*)] \frac{b}{4} \delta f(k); \quad (1.2)$$

he takes a position

$$\alpha^* = \frac{b}{2} \quad (1.3)$$

and engages in managerial disciplining. Otherwise the activist does not participate.

The market maker, upon observing net order flow ω , sets the following prices

$$\begin{aligned}
P(\omega) = P_l &\equiv \left[\frac{(1-z)+z(1-\lambda G(c_t^*))(1-\delta)}{1-z\lambda G(c_t^*)} \right] f(k) & \text{if } \omega < -b + \alpha^* \\
P(\omega) = P_m &\equiv [1 - z(1 - \lambda G(c_t^*))\delta] f(k) & \text{if } \omega \in [-b + \alpha^*, 0] \\
P(\omega) = P_h &\equiv f(k) & \text{if } \omega > 0.
\end{aligned} \tag{1.4}$$

A full proof is in the Appendix; here we provide the key intuition. After observing the net order flow, the market maker updates beliefs according to Bayes rule and sets prices in (1.4). Letting a_1 denote activism and a_0 denote the absence of activism, the market maker's pricing rule satisfies

$$\begin{aligned}
P &= E[V|\omega] \\
&= \left[\Pr[a_1|\omega] \Pr[V = f(k) | a_1] + \Pr[a_0|\omega] \Pr[V = f(k) | a_0] \right] f(k).
\end{aligned} \tag{1.5}$$

When the net order flow is sufficiently negative, the market maker knows with certainty that the activist did not buy shares, i.e., $\Pr[a_1|\omega < -b + \alpha^*] = 0$. The market maker infers that either (a) there was no malfeasance (which happens with unconditional probability $1 - z$), in which case the project pays $f(k)$; or (b) there was malfeasance, and either the malfeasance was not discovered, or the cost of intervention was too high (which collectively happen with unconditional probability $z[1 - \lambda G(c_t^*)]$), in which case expected project payoffs are $(1 - \delta)f(k)$. Similarly, a positive net order flow reveals that the activist took a position with certainty, in which case the project is sure to pay off $f(k)$: $\Pr[a_1|\omega > 0] = 1$ and $\Pr[V = f(k) | a_1] = 1$. In contrast, intermediate net order flows $\omega \in [-b + \alpha^*, 0]$ are consistent with both the presence and the absence of activism, and allow the activist to extract information rents from uninformed investors. Given a net order flow of $\omega \in [-b + \alpha^*, 0]$, the market maker knows that either liquidity trade was $l = -\omega$, or that liquidity trade was $l = -\omega + \alpha^*$. With the uniform distribution, these densities cancel out of the numerator and denominator of the conditional probability of activism, causing

the market maker to set price equal to the *unconditional* expected project payoff regardless of the level of net order flow $\omega \in [-b + \alpha^*, 0]$. It is this feature of the uniform distribution that simplifies analysis.

When the activist participates and liquidity shocks outweigh the number of shares that he buys, i.e., $l \geq \alpha^*$, the activist acquires the stock below its true value at $P_m < f(k)$. If, instead, $l < \alpha^*$, then the activist pays P_h for each share and makes no profit. The probability that the activist camouflages his share purchase with liquidity sales is $\int_{\alpha}^b \frac{1}{b} dl = \frac{b-\alpha}{b}$. It follows that his expected gross profits conditional on buying α shares are

$$E[\Pi_A | a_1] = \left(\frac{b-\alpha}{b} \right) \alpha [f(k) - P_m]. \quad (1.6)$$

Inspection of (1.6) reveals that the activist faces a trade-off between the number of undervalued shares that he may acquire α and the expected cost of information revelation $\frac{b-\alpha}{b}$. This captures adverse price effects by which the expected stock price paid by the activist rises as he buys more shares. The activist's expected trading profits in (1.6) are maximized by a share purchase of $\alpha^* = b/2$. Greater liquidity b makes it easier for the activist to camouflage his trade, encouraging him to acquire a larger position.¹⁵

If the activist observes managerial malfeasance, he disciplines management when doing so is expected to be profitable, i.e., when $E[\Pi_A | a_1] \geq c$. This relation pins down the activist's cost participation cut-off in equilibrium:

$$c_t = z [1 - \lambda G(c_t)] \left(\frac{b-\alpha}{b} \right) \alpha \delta f(k), \quad (1.7)$$

which takes the form in (1.2) when evaluated at the optimal position of $\alpha^* = b/2$,

¹⁵The positive relation between informed trading and market liquidity has long been studied in the literature. See [Bond et al. \(2012\)](#)'s review of the role of liquidity in the real effects of financial markets, and the review by [Edmans and Holderness \(2016\)](#) of the effect of liquidity in the context of blockholder interventions.

i.e., $c_t^* \equiv c_t(\alpha^*)$.¹⁶ The cut-off c_t is unique and maximized for $\alpha = \alpha^*$. In equilibrium, the activist employs a threshold strategy such that, conditional on observing malfeasance, he buys α^* shares and disciplines management if and only if $c \leq c_t^*$.¹⁷

The endogenous cut-off c_t^* captures two key equilibrium features. First, it represents the activist participation threshold, and thus the extent of managerial disciplining. The probability that the activist intervenes to discipline the manager after observing the manager taking an action that reduces shareholder value is $G(c_t^*)$. Thus, a higher c_t^* implies superior governance. Second, c_t^* captures the activist's expected conditional trading profits. In equilibrium, the activist's expected trading profits *equal* the expected trading losses of uninformed investors because trading is a zero-sum game in which the market maker expects to break even. Thus, c_t^* represents the expected transfer of trading profits from uninformed investors to the activist conditional on the activist intervening.

These conditional trading transfers c_t^* increase with investment k . The greater is real investment, the greater is the project value, and hence (i) the more valuable is managerial disciplining, and (ii) the more profitable it is for the activist to intervene. Two assumptions drive this result. First, the cost of activism is independent of the company's value, so the incentives for disciplining are positively related to stock ownership (Shleifer and Vishny 1986).¹⁸ Second, because the value enhanced by the intervention is multiplicative, rather than additive, the relevant measure of incentives is the activist's *dollar* ownership, not its *share* ownership (Edmans and Holderness 2016).¹⁹

Conditional trading transfers c_t^* also rise with market liquidity b . High liquid-

¹⁶To obtain (1.7) set $c_t = E[\Pi_A|a_1]$ using the expression in (1.6) substituting for the price P_m using (1.4).

¹⁷To verify uniqueness note that the left-hand side of (1.7) increases in c_t and the right-hand side decreases.

¹⁸Brav et al. (2016) argue that it can be harder for activists to intervene in larger companies due to credit constraints. Our model can be modified to provide a similar prediction in the presence of financial constraints.

¹⁹In the related context of CEO incentives, Baker and Hall (2004) and Edmans et al. (2009) show theoretically that a CEO's dollar ownership and not percentage ownership is relevant when the CEO has a multiplicative effect on firm value.

ity increases activist trading profits and thus the probability $G(c_t^*)$ that the activist finds it profitable to discipline management. In line with [Kahn and Winton \(1998\)](#) and [Maug \(1998\)](#), higher liquidity allows the activist to increase his position with a reduced risk of discovery, thereby encouraging intervention. [Back et al. \(2017\)](#) model the dynamics of position building by activist funds and show the potentially positive effects of liquidity. Consistent with this, [Collin-Dufresne and Fos \(2015\)](#) and [Gantchev and Jotikasthira \(2017\)](#) provide evidence that activist funds camouflage their purchases with liquidity trades by other parties.

Investment

At $t = 0$ uninformed investors anticipate trading outcomes and activism levels, and invest capital so as to maximize expected profits. In addition to the investment decision, Proposition 2 characterizes the expected project payoffs and how they are split among market participants in expectation at $t = 0$. This sets the ground for the analysis of the main interacting forces in the model and the introduction of blockholder disclosure thresholds.

Proposition 2 *The expected value at $t = 0$ of the project given investment k is*

$$E[V] = [1 - z(1 - \lambda G(c_t^*))\delta]f(k) \equiv \pi_V f(k). \quad (1.8)$$

The expected gross profits of the activist are:

$$E[\Pi_A] = z\lambda G(c_t^*) \frac{c_t^*}{f(k)} f(k) \equiv \pi_A f(k). \quad (1.9)$$

The expected gross profits of uninformed investors are:

$$E[\Pi_I] = (\pi_V - \pi_A)f(k) \equiv \pi_I f(k). \quad (1.10)$$

The investment k by uninformed investors solves

$$\pi_I f'(k) - r = 0. \quad (1.11)$$

Total expected cash flows are the product of $f(k)$ and the probability that the project succeeds $\pi_V \in [0, 1]$. Proposition 2 reveals that expected total rents are split between the activist and uninformed investors in proportions π_A/π_V and π_I/π_V respectively. This follows because the market maker earns zero expected profits, which means that activist trading profits are extracted one-for-one from uninformed investors. More formally, the expected gross profits of the activist are captured by the product of the unconditional probability that he participates $z\lambda G(c_t^*)$ and the expected trading profits from participating c_t^* . Uninformed investors obtain, in expectation, the rest of the “pie”, $(\pi_V - \pi_A) f(k)$. Real investment, characterized by (1.11), maximizes expected profits of uninformed investors at date 0.

Proposition 2 shows that activism has an impact on real investment via its effect on the expected profits of uninformed investors. Investors face a tension as to their preferred extent of activism, where the extent of activism is captured by $G(c_t^*)$. Higher transfers of trading profits c_t^* increase the proportion of cash flows taken by the activist in expectation π_A , reducing the investors’ portion π_I . However, higher trading transfers also incentivize activist participation, and the increased managerial discipline raises total expected cash flows $\pi_V f(k)$. As a result, greater trading transfers c_t^* to activists need not hurt uninformed investors. In particular, activism fosters real investment when investor gains from managerial disciplining outweigh the associated trading losses, and it discourages real investment otherwise.

This mechanism underscores the *investment feedback effect* faced by the activist. The value of activism is directly related to the size of the project—the profitability of the activist grows with real investment, i.e., c_t^* grows with k . But, expected levels of activism affect investment. Therefore, expected activism affects

real investment, which, in turn, affects the extent of activism. Crucially, the activist does not internalize this investment feedback in his trading decision because this is taken at $t = 2$, when real investment has already been sunk. Thus, when the activist participates, he takes a position α^* to maximize *conditional* expected profits (1.6), i.e., for a given k , rather than *unconditional* expected profits (1.9).

Our analysis identifies novel strategic interactions between uninformed investors and activist funds. The linkage between investment and trading profit transfers is similar to that found in papers studying the real effects of informed trading (Leland 1992; Bernhardt et al. 1995). We incorporate a new element: the informed trader is an activist fund who can increase investment value by alleviating agency problems between owners and managers (Brav et al. 2008; Klein and Zur 2009, Brav et al. 2015; Bebchuk et al. 2015). The effect of hedge fund activism on real investment is thus twofold: Informed trading reduces the profitability of uninformed investors who respond with lower investment; but it also encourages the intervention of activist funds that discipline management, thereby incentivizing investment.

1.3 Blockholder Disclosure Thresholds

Blockholder disclosure thresholds are rules that require a shareholder to disclose stock holdings when they reach a certain fraction of the overall voting rights in a publicly traded firm. In recent years, hedge fund activism has led some market participants and commentators to call for an expansion of these rules, and policy makers are now contemplating how to respond. We next briefly describe the institutional framework. We then contribute to the regulatory debate by deriving the optimal threshold policies for investors, hedge fund activists and society.

1.3.1 Institutional Framework and Regulatory Debate

A key concern for financial regulators is the protection of minority shareholders against dominant shareholders. Ownership disclosure rules are considered to be one such protection mechanism. The OECD’s Principles of Corporate Governance of 2004 advocate that “one of the basic rights of investors is to be informed about the ownership structure of the enterprise” (OECD, 2004). In the US, the disclosure of “beneficial ownership” is regulated by the Williams Act of 1968, passed in response to a wave of hostile takeover attempts, mostly through tender offers.²⁰ In Europe, the EU Transparency Directive of 2004 claims that the disclosure of major holdings in listed companies should enable investors to acquire or dispose of shares in full knowledge of changes in the voting structure (EC, 2004). Along with other corporate transparency rules, blockholder disclosure thresholds are set to prevent the expropriation of rents by large shareholders that gain influence or control of their companies at the expense of uninformed investors.²¹

Investor protection is key to increasing confidence and encouraging real investment. For instance, the EU Transparency Directive belongs to a range of measures that aim, among other things, to enable issuers to raise capital on competitive terms across Europe (Schouten 2010). La Porta et al. (2006) and Djankov et al. (2008) provide evidence that greater legal protection of investors is associated with more developed financial markets. Both studies construct protection indices that include ownership disclosure rules. Our paper captures the link between investor protection and investment, but it challenges the extended view on the relationship between corporate transparency and protection. Our earlier analysis shows that the ability to secretly acquire stock encourages activist funds to discipline management, alleviating agency problems between uninformed investors and managers, consistent with the empirical evidence that hedge fund activists enhance the value of their

²⁰See Nagel et al. (2011) for a more extensive analysis and more detailed arguments.

²¹See Edmans and Holderness (2016) for a review of the literature on the costs imposed by blockholders that pursue their own private benefits.

target companies.

Blockholder disclosure thresholds differ across financial systems. For example, investors that intend to introduce corporate changes in US publicly-listed companies must fill a 13(d) file when their holdings reach 5% of voting rights. In Canada, equivalent disclosure of ownership is not required until a 10% stake is acquired. In the EU, Germany recently reduced the threshold to 3%, which is also the cutoff in the UK, while the threshold in France remains at 5%. Regulation across jurisdictions also differs in such elements as the time window to report acquisitions, the information that must be disclosed, and the types of securities subject to disclosure.²² Schouten and Siems (2010) identify a historical trend and convergence towards greater ownership disclosure, i.e., towards lower thresholds. Yet, despite the vast potential impacts of small differences in these rules, it remains unclear what brings regulatory bodies to set a disclosure threshold of, for instance, 5% rather than 2% or 10%; and Edmans and Holderness (2016) ask why regulators (and researchers) tend to focus on measures of percentage ownership and neglect those of absolute ownership.

The increasing importance of hedge fund activism has altered the regulatory debate. Broadly, ownership disclosure rules were set to inform investors about stock acquisitions that can result in takeovers or proxy fights. The WLRK (2011) Petition argued that these rules no longer serve their purpose because activist funds can gain control of target companies with small positions. Several academics opposed the petition arguing that hedge fund activism increases the value of target companies and hence has positive externalities for other investors. According to Bebchuk and Jackson (2012) this was at the heart of the Williams Act, which considered that outside investors who acquire large blocks of stock “should not be discouraged, since they often serve a useful purpose by providing a check on entrenched but inefficient

²² “Stakebuilding, mandatory offers and squeeze-out comparative table” by Practical Law, of Thomson Reuters provides a synthesis of ownership disclosure rules in financial systems. External link to the table [here](#).

management.” The debate is not confined to the US. For instance, in Canada, a recent proposal to reduce the disclosure threshold from 10% to 5% was opposed by two major associations of investment funds that argued “the lower threshold will make share acquisitions by engaged investors more expensive and, in many circumstances, too costly to justify the resources, time and effort for such activity. This, in turn, will chill the market for engaged investing, and erode the benefits of the value creation that results from having shareholder engagement” (MFA and AIMA 2013).

1.3.2 Optimal Policies

We now extend our model to show how blockholder disclosure thresholds can regulate the level of hedge fund activism, deriving the optimal policies for investors, activist and society.

Ownership disclosure rules may limit the number of undervalued shares that the activist can acquire, reducing his incentives to participate. If a legal disclosure threshold $\bar{\alpha}$ is implemented, an activist must publicly announce his position when it crosses the threshold. Then the activist has no incentive to establish a larger position because doing so would reveal his presence causing the stock price to rise to $P_h = f(k)$, which would eliminate his information rents, rendering intervention unprofitable.²³ Corollary 3 follows immediately:

Corollary 3 *A disclosure threshold $\bar{\alpha}$ is binding if and only if $\bar{\alpha} < \alpha^*$. In equilibrium, when a disclosure threshold binds the activist sets $\alpha = \bar{\alpha}$.*

The activist’s conditional trading profits $c_t(\alpha)$ in (1.7) increase with his position for $\alpha < \alpha^*$. Thus, when the activist participates, he acquires a position $\alpha = \min \{\bar{\alpha}, \alpha^*\}$. The mechanism implies that for a given firm characterized by $f(k)$, a binding threshold necessarily reduces both the profits and extent of hedge fund

²³This price reaction is consistent with evidence by [Bebchuk et al. \(2015\)](#) that the stock-price spike that follows disclosure reflects the long-term value of the intervention.

activism. To see this, let \bar{c}_t represent the trading profits, and hence participation cut-off, associated with a position determined by a binding threshold $\bar{\alpha} < \alpha^*$. Because trading profits increase in α , activism is now less profitable, i.e., $\bar{c}_t < c_t^*$, making the activist less likely to participate, i.e., $G(\bar{c}_t) < G(c_t^*)$.²⁴ A direct consequence is that managerial malfeasance is more likely to destroy value. This mechanism is consistent with arguments against expanding ownership disclosure rules (see Section 3.1). However, our paper shows that they only comprise part of the overall effect.

The argument is incomplete because it neglects the effects of a disclosure threshold on real investment. Changes in expected levels of activism at $t = 2$ also alter real investment at $t = 0$, which, in turn, affects the activist's incentives to participate. A binding disclosure threshold reduces the conditional transfer of trading profits from investors to the activist, which may incentivize real investment, creating a positive investment feedback that can increase activism.

Proposition 4 derives the consequences of blockholder disclosure thresholds by characterizing the ordering of the optimal disclosure threshold policies for investors, the activist and a welfare-maximizing regulator representing society. We denote these policies $\bar{\alpha}_I$, $\bar{\alpha}_A$ and $\bar{\alpha}_R$ respectively. We present our results as a function of the *profit elasticity of activism*,

$$\varepsilon_a(c_t) = \frac{\partial G(c_t)}{\partial c_t} \frac{c_t}{G(c_t)}.$$

Here, ε_a captures the responsiveness of activism to informed trading: the higher is ε_a , the bigger is the increment in the probability that the activist intervenes $G(c_t)$ in response to a marginal increase in expected trading profits c_t . Absent a binding disclosure threshold, when the activist participates he buys α^* shares, earns expected gross profits c_t^* , and the profit elasticity of activism is $\varepsilon_a(c_t^*) \equiv \varepsilon_a^*$.

Proposition 4 *There exists cutoffs $\varepsilon_a^{*R} \equiv -\left(\frac{c_t^*}{\delta f(k) - c_t^*}\right) \left[\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t^*/d\alpha}{c_t^*}}\right]$ and $\varepsilon_a^{*I} \equiv \left(\frac{c_t^*}{\delta f(k) - c_t^*}\right)$,*

²⁴It follows from the analysis in Section 2 that $\bar{c}_t = z[1 - \lambda G(\bar{c}_t)]\left(\frac{b - \bar{\alpha}}{b}\right)\bar{\alpha}\delta f(k) < c_t^*$.

on the profit elasticity of activism where $\varepsilon_a^{*R} < \varepsilon_a^{*I}$ such that

1. No one benefits from a binding disclosure threshold if the profit elasticity of activism is sufficiently high: $\varepsilon_a^* \geq \varepsilon_a^{*I} \Rightarrow \alpha^* \leq \{\bar{\alpha}_I, \bar{\alpha}_A, \bar{\alpha}_R\}$.
2. Only investors benefit from a binding disclosure threshold if the profit elasticity of activism is intermediate: $\varepsilon_a^{*R} \leq \varepsilon_a^* < \varepsilon_a^{*I} \Rightarrow 0 < \bar{\alpha}_I < \alpha^* \leq \{\bar{\alpha}_A, \bar{\alpha}_R\}$.
3. Both investors and society gain from a binding disclosure threshold if the profit elasticity of activism is low, with investors gaining more: $\varepsilon_a^* < \varepsilon_a^{*R} \Rightarrow 0 < \bar{\alpha}_I < \bar{\alpha}_R < \alpha^* \leq \bar{\alpha}_A$.

Figure 1.2 illustrates the results; a full proof is in Appendix A. Optimal disclosure threshold policies are characterized by the first order conditions (FOCs) of net profit functions with respect to the activist position α . Corollary 3 implies that when the optimal position is less than α^* , it can be achieved in equilibrium by a binding disclosure threshold.

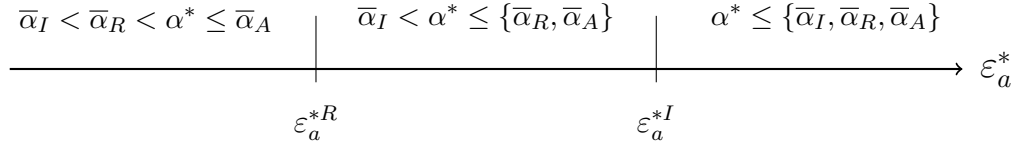


Figure 1.2: Optimal Disclosure Thresholds

Uninformed investors maximize $\pi_I f(k) - rk$. The associated FOC reveals that they benefit from a binding disclosure threshold if and only if

$$g(c_t^*) [\delta f(k) - c_t^*] < G(c_t^*), \quad (1.12)$$

which can be rearranged to $\varepsilon_a^* < \varepsilon_a^{*I}$. The left-hand side (LHS) represents the marginal benefits to uninformed investors of increasing the transfer of trading profits to the activist when $\alpha = \alpha^*$, i.e., for c_t^* . Higher transfers cause the probability that the activist participates conditional on observing managerial malfeasance to rise

by $g(c_t^*)$. The associated benefit for investors is the difference between the total value enhanced by the activist $\delta f(k)$ and their trading losses c_t^* . The right-hand side (RHS) captures the conditional loss from marginally higher transfers: with probability $G(c_t^*)$ the activist would have participated anyway, even if expected trading profits had not increased.

A binding disclosure threshold $\bar{\alpha}$ reduces transfers of trading profits, $c_t(\bar{\alpha}) \equiv \bar{c}_t < c_t^*$. (1.12) shows that this raises the marginal benefits to investors of activism (LHS) and reduces the associated losses (RHS), increasing marginal profitability. Equivalently, a binding threshold increases the profit elasticity of activism ε_a , and it requires less trading transfers from investors to encourage higher activism. Transfers of trading profits are the cost that investors incur in exchange for managerial discipline, and this cost rises with the extent of activism.

The optimal extent of activism for investors solves this FOC: $\bar{\alpha}_I$ solves $\varepsilon_a = c_t / [\delta f(k) - c_t]$ when $\varepsilon_a^* < \varepsilon_a^{*I}$. Full disclosure is never optimal. If the activist cannot acquire stock secretly, trading profits and hence transfers vanish, i.e., if $\alpha \rightarrow 0$ then $c_t \rightarrow 0$. But then the activist never participates. Then, the marginal benefits of discipline for investors outweigh the corresponding trading losses, i.e., $g(c_t) [\delta f(k) - c_t] > G(c_t)$. Thus, uninformed investors always benefit from some degree of market opacity, i.e., $\bar{\alpha}_I > 0$: the marginal profitability to uninformed investors of activism is always positive whenever $\bar{\alpha}_I$ is sufficiently small.

The optimal extent of activism can be achieved by a disclosure threshold when the corresponding trading transfers are lower than those in the unconstrained equilibrium, i.e., when (1.12) holds, but not otherwise. The mechanism highlights the asymmetric role of disclosure rules, which can only limit, but not foster, informed trading. If, absent regulation, the marginal profitability of activism for investors is positive, i.e., if (1.12) is not satisfied, the desired extent of activism cannot be achieved and the optimal policy is non-binding, i.e., $\bar{\alpha}_I \geq \alpha^*$. We discuss below the role of market liquidity, which determines α^* and thus whether a particular

disclosure threshold is binding.

Our argument builds on the result that transfers of trading profits increase with the activist's position, i.e., $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$. It follows that restricting α reduces c_t . *This is not immediate.* We earlier established that the activist faces an investment feedback effect that he does not internalize. In particular, the activist's position at $t = 2$ influences initial investment k , and this determines the trading profits from a given position α . A binding disclosure threshold regulates the number of shares that the activist buys in equilibrium. We have

$$\frac{dc_t}{d\alpha} = \underbrace{\frac{\partial c_t}{\partial \alpha}}_{\text{mg. net trading transfer}} + \underbrace{\frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \alpha}}_{\text{investment feedback}}. \quad (1.13)$$

Net trading transfers capture the effect of the activist's position on transfers at $t = 2$ for a given investment k . (1.7) revealed that $\frac{\partial c_t}{\partial \alpha} > 0 \Leftrightarrow \alpha < \alpha^*$, leading the activist to take a position α^* in the absence of a disclosure threshold (Proposition 1). The investment feedback effect captures the impact of the activist's position on real investment $\frac{\partial k}{\partial \alpha}$, and the effect that it has, in turn, on trading transfers $\frac{\partial c_t}{\partial k}$. Real investment always raises trading transfers, and thus the extent of activism, i.e., $\frac{\partial c_t}{\partial k} > 0$. However, the activist position α might be large enough to hurt investors, who respond by reducing investment. That is, if $\alpha > \bar{\alpha}_I$ then $\frac{\partial k}{\partial \alpha} < 0$, and the effect of a larger position on trading transfers is determined by the balance of two opposing forces: positive net transfers and a negative investment feedback. We show that, surprisingly, the tension is always resolved against the investment feedback effect, so $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$.

This result reflects the subordinated nature of investment feedback with respect to the direct impact of trading transfers. Intuitively, these transfers lead the activist to take a position α^* , which, in turn, affects investment. If the reduction of investment from increasing α was strong enough to reduce the activist's trading profits, i.e., if $\frac{dc_t}{d\alpha} < 0$, it would also increase investor profits because

$g(c_t) [\delta f(k) - c_t] < G(c_t)$ when $\frac{\partial k}{\partial \alpha} < 0$. But then investors would *increase* investment, not reduce it, benefiting activists. Since trading transfers are the activist's sole source of income, this mechanism explains why he never benefits from a binding disclosure threshold:

Corollary 5 *Negative investment feedback reduces the positive impact of increasing the activist position on trading profits c_t and thus on the extent of activism $G(c_t)$. However, it does not alter the sign of the impact, i.e., $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$: the activist never benefits from a blockholder disclosure threshold just because it boosts investment.*

Thus, when investors seek a binding disclosure threshold $\bar{\alpha}_I < \alpha^*$, a conflict of interest arises between them and the activist. An activist position that exceeds $\bar{\alpha}_I$ harms investors, reducing real investment. This, in turn, reduces the profitability of activism and the levels of managerial discipline (negative investment feedback). Nonetheless, the investment response is never strong enough to outweigh the net positive effect of additional shares on activist profits. Therefore, the activist never wants a binding disclosure threshold to increase investment.

Society maximizes total expected value net of the costs of capital rk and the expected costs of activism $z\lambda G(c_t) E[c|c \leq c_t]$. Society gains from a binding disclosure threshold if

$$z\lambda g(c_t^*) [\delta f(k) - c_t^*] \frac{dc_t^*}{d\alpha} + \pi_A f'(k) \frac{\partial k}{\partial \alpha} < 0, \quad (1.14)$$

which can be rearranged to $\varepsilon_a^* < \varepsilon_a^{*R}$.²⁵ The condition reveals that society cares about both the value-enhancing effects of activism and real investment. The first term in (1.14) captures the impact of the activist's equity position on project value via managerial discipline. This is positive for all $\alpha < \alpha^*$. In particular, Corollary 5 shows that the extent of activism is directly related to the activist's position

²⁵See Appendix A. Rearrangement of (1.14) to obtain $\varepsilon_a^* < \varepsilon_a^{*R}$ uses $\frac{df(k)}{d\alpha} = f'(k) \frac{\partial k}{\partial \alpha}$.

regardless of the investment feedback, i.e., $\frac{dc_t}{d\alpha} > 0$. Moreover, greater managerial discipline always creates value. Here, $g(c_t) \delta f(k)$ is the conditional increase in gross value, and $g(c_t) c_t$ is the corresponding increase in expected cost of activism. The second term in (1.14) represents investment feedback that is not internalized by investors. More specifically, real investment solves $\pi_I f'(k) - r = 0$, but the optimal investment for society sets $(\pi_I + \pi_A) f'(k) - r = 0$,

Society only benefits from a disclosure threshold if investors also do so, but the converse is not true. For $\varepsilon_a^* < \varepsilon_a^{*R}$ to hold, the investment feedback must be negative, i.e., $\frac{\partial k}{\partial \alpha} < 0$, implying that $\varepsilon_a^* < \varepsilon_a^{*I}$. Intuitively, society only cares about the real economy, and not about secondary markets (trading transfers). The only social cost of increasing managerial disciplining is the potential reduction in investment. If this is sufficiently strong, then (1.14) holds and the regulator wants to set a binding disclosure threshold. Still, this threshold always exceeds the optimal threshold from the perspective of investors who do care about trading transfers.

That society's preferred disclosure threshold lies (weakly) between those preferred by investors and activists also arises in models of insider trading where real investment is endogenous (Leland 1992; Bernhardt et al. 1995). The social cost of increasing activism is a potential reduction of real investment due to lower market transparency. Thus, a regulator only considers implementing a disclosure threshold when this also benefits investors. Yet, while investors incur trading losses society does not, so the socially optimal level of market transparency is lower. Therefore, when the regulator seeks a binding threshold, it always exceeds the preferred threshold of investors, and sometimes they disagree on the need for a binding policy.

The role of liquidity and the cost of activism

Results in Proposition 4 are influenced by the interaction of two opposing forces: (i) market liquidity and (ii) the cost of activism. We discuss and interpret their role.

Corollary 6 *Let a cost distribution function g be such that no one benefits from*

a binding disclosure threshold, i.e., $\varepsilon_a^* \geq \varepsilon_a^{*I}$. Reduce the cost of activism with a transfer of probability mass $\min\{g(c), \tau\}$ from each realization $c \in (0, C]$ to $c = 0$. There exist cutoffs $\tau^I(b) < \tau^R(b)$ such that

1. Investors gain from a binding disclosure threshold if and only if $\tau > \tau^I(b)$.
2. Society gains from a binding disclosure threshold if and only if $\tau > \tau^R(b)$.

Both cutoffs decrease with market liquidity, i.e., $\tau^{I'}(b) < 0$ and $\tau^{R'}(b) < 0$.

Corollary 6 considers the effect of a reduction in the cost of activism consisting of an even decrease in the probability of any positive cost. Transformation $\tau > 0$ scales down density g , reducing both the expected cost and profit elasticity of activism ε_a . As a result, the marginal profitability of trading transfers for investors is less at any given c_t , and they benefit from a binding disclosure threshold when the cost reduction is large enough. In particular, $\varepsilon_a^* = \varepsilon_a^{*I}$ when $\tau = \tau^I(b)$, and a larger τ tightens the optimal threshold. Analogous intuition holds for society. At the limit, as τ grows arbitrarily large, activism becomes almost costless, and investors do not need to incentivize activist participation with higher trading profits. Then, the optimal policy for both investors and society approaches full transparency, i.e., $\{\bar{\alpha}_I, \bar{\alpha}_R\} \rightarrow 0$.

Both cutoffs $\tau^I(b)$ and $\tau^R(b)$ decrease with the extent of market liquidity b because this makes activism more profitable: α^* and c_t^* both rise. All else equal, this reduces the profit elasticity of activism, and may make a binding disclosure beneficial for investors. In (1.12), the LHS decreases and the RHS increases. It follows that the cost reduction that leads investors to gain from a binding policy falls as market liquidity rises.

These results are intuitive. The activist requires market liquidity to establish an equity stake and profit from intervention (Maug 1998; Kahn and Winton 1998). Disclosure thresholds operate against liquidity by increasing market transparency

and limiting the activist's position (Bebchuk et al. 2013). Greater liquidity reduces the marginal profitability of activism for investors, making a disclosure threshold more desirable. The cost of activism operates in the opposite direction. When the cost of managerial discipline is likely to be high, investors want to concede further trading transfers to incentivize activism, and they do not benefit from a disclosure threshold. In particular, with high costs, i.e., big g , the profit elasticity of activism is large for relatively opaque markets, and investors do not want to limit the potential trading profits of activists.

The cost of activism is often related to managerial entrenchment. Staggered boards make it harder to gain control of a company in a proxy contest, discouraging activism. Our model is consistent with Gompers et al. (2003) and Bebchuk and Cohen (2005), who find evidence of a negative correlation between firm value and management-favouring provisions. In such instances, relaxing disclosure thresholds can benefit investors by alleviating the negative effect of these provisions at the expense of market opacity and investor trading losses.

1.4 Managerial Feedback

We next endogenize the probability of managerial malfeasance to characterize the complete inter-linkage between investment, hedge fund activism and corporate management, and the consequences for optimal blockholder disclosure threshold policies.

We extend our model by assuming that if the manager implements the good business plan ($m = 1$) at $t = 1$, she receives a payoff that is normalized to zero at $t = 3$. If, instead, the manager adopts the bad plan ($m = 0$), her payoff depends on whether she is disciplined by the activist. When the activist does not intervene, adopting the bad business plan gives the manager a fixed benefit φ . If the activist disciplines the manager, she does not receive the private benefit and incurs a privately-observed reputation cost $\rho > 0$. Other market agents share a common

prior that ρ is distributed on $[0, R]$ according to a strictly positive density h and associated cumulative function H . We assume that private benefits from malfeasance are not too high: $\varphi \leq 4R$.²⁶

Both private benefits from malfeasance φ and the reputation costs of being disciplined by an activist ρ allow for multiple interpretations.²⁷ For instance, managerial benefits from acting against shareholders might be related to increasing executive compensation or empire-building mergers and acquisitions that managers value but harm firm value. The costs of being disciplined by an activist may reflect career prospects. For example, Fos and Tsoutsoura (2014) report that facing a direct threat of removal is associated with \$1.3-\$2.9 million in foregone income until retirement for the median incumbent director in their sample; and Keusch (2017) finds that in the year after activists intervene, internal CEO turnover rises 7.4%.

1.4.1 Market Equilibrium

The manager employs a threshold strategy, implementing the bad business plan if and only if $\rho \leq \rho_t$. At the cut-off, the expected private benefits from malfeasance equal the expected loss due to punishment, $(1 - \lambda G(c_t))\varphi = \lambda G(c_t)\rho_t$, which we solve for:

$$\rho_t = \varphi \left[\frac{1 - \lambda G(c_t)}{\lambda G(c_t)} \right]. \quad (1.15)$$

The probability of managerial malfeasance is $H(\rho_t)$. The equilibrium analysis is analogous to Section 2.1, where both Propositions 1 and 2 extend by directly setting $z \equiv H(\rho_t)$.

The solution for ρ_t reveals that malfeasance declines with the *conditional* probability of activism $G(c_t)$: the more likely the activist is to participate after observing malfeasance, the less likely is the manager to misbehave. We call the man-

²⁶We use this upper bound on the private benefits from malfeasance φ to ease establishing that second-order conditions hold in the derivation of optimal blockholder disclosure threshold policies.

²⁷Because the firm's manager only cares about the *net* benefit of malfeasance, one could alternatively assume that the benefits of malfeasance are random, and that the reputation cost is fixed.

agers’ response to the threat of activism, the *managerial feedback* effect. This effect is negative, reflecting that the threat of activism deters managers from destroying shareholder value. Activism disciplines management through two complementary channels: (i) ex post, the activist intervenes to change the business plan when it is bad; (ii) ex ante, it discourages the adoption of the bad plan.

The mechanism is consistent with anecdotes suggesting that executives of firms that are yet-to-be-targeted by activist funds feel threatened and proactively work with advisors and lawyers to evaluate firm policies that minimize the vulnerability to attacks by activist funds.²⁸ More formally, [Gantchev et al. \(2017\)](#) find evidence that non-target firms, observing that their peers are being targeted by activists, perceive a higher risk of becoming a future target, and change their policies to mitigate this risk.

1.4.2 Optimal Policies

We study optimal blockholder disclosure threshold policies when the probability of managerial malfeasance is endogenous. Managerial feedback raises new policy questions. For example, additional trading profits increase the conditional profitability of activism (Proposition 4) and reduce activists’ opportunity to profit (managerial feedback). Do investors still benefit from a disclosure threshold? What are the implications for real investment, and thus for society? How does managerial feedback affect the reluctance of hedge fund activists to support ownership disclosure rules? Could activist funds seek lower thresholds than investors?

Proposition 7 answers these questions, characterizing the ordering of optimal disclosure policies for market participants. We define an elasticity measure that

²⁸See, for instance, “Key Issues for Directors in 2014” by Martin Lipton of Wachtell, Lipton, Rosen and Katz, The Harvard Law School Forum on Corporate Governance and Financial Regulation, December 16, 2013; “Boardrooms Rethink Tactics to Defang Activist Investors”, The New York Times, November 11, 2013.

allows us to present results intuitively: the *activism elasticity of management*,

$$\varepsilon_m = \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{G(c_t)}{H(\rho_t)}.$$

Here, $\varepsilon_m < 0$ captures a manager's reaction to the threat of activism. The bigger is ε_m (in absolute value), the larger is the reduction in the probability of managerial malfeasance $H(\rho_t)$ in response to a marginal increase in the conditional probability of activism $G(c_t)$. In the absence of a binding disclosure, when the activist participates he buys α^* shares and has expected gross profits c_t^* . Moreover, the manager adopts the bad business plan if and only if $\rho \leq \rho_t(c_t^*) \equiv \rho_t^*$ and the activism elasticity of management is $\varepsilon_m(c_t^*, \rho_t^*) \equiv \varepsilon_m^*$. In the proposition below, we assume that second-order conditions are well-behaved for investors and activists; the Appendix shows that this will be so when the costs of intervention for the activist and the reputation costs of management have uniform distributions.

Proposition 7 *Suppose that the net expected profits of investors and activists are quasiconcave in α for $\alpha \leq \alpha^*$. Then there exist cutoffs on the activism elasticity of management, $\varepsilon_m^{*A} \equiv -\frac{1}{\varepsilon_a^*} \left(\frac{c_t^*}{c_t^* - E[c|c \leq c_t^*]} \right)$, $\varepsilon_m^{*I} \equiv -\left(\frac{\partial \pi_A / \partial H(\rho_t^*)}{\partial \pi_I / \partial H(\rho_t^*)} \right) \left[\frac{\delta f(k) - c_t^*}{c_t^*} - \frac{1}{\varepsilon_a^*} \right]$ and $\varepsilon_m^{*R} \equiv \left[\frac{\delta f(k) - c_t^*}{c_t^*} + \frac{1}{\varepsilon_a^*} \left(\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t^*/d\alpha}{c_t^*}} \right) \right] \left[-\frac{\partial \pi_I / \partial H(\rho_t^*)}{\partial \pi_A / \partial H(\rho_t^*)} - \frac{c_t^* - E[c|c \leq c_t^*]}{c_t^*} \right]^{-1}$, where $\varepsilon_m^{*A} < \varepsilon_m^{*I} < \varepsilon_m^{*R}$ such that*

1. *If the activism elasticity of management is sufficiently high, then only the activist benefits from a binding disclosure threshold: $\varepsilon_m^* < \varepsilon_m^{*A} \Rightarrow 0 < \bar{\alpha}_A < \alpha^* \leq \{\bar{\alpha}_I, \bar{\alpha}_R\}$.*
2. *If the activism elasticity of management is moderately high, then no one benefits from a binding disclosure threshold: $\varepsilon_m^{*A} \leq \varepsilon_m^* \leq \varepsilon_m^{*I} \Rightarrow 0 < \alpha^* \leq \{\bar{\alpha}_I, \bar{\alpha}_A, \bar{\alpha}_R\}$.*
3. *If the activism elasticity of management is moderately low, then only investors benefit from a binding disclosure threshold: $\varepsilon_m^{*I} < \varepsilon_m^* \leq \varepsilon_m^{*R} \Rightarrow 0 < \bar{\alpha}_I < \alpha^* \leq$*

$$\{\bar{\alpha}_A, \bar{\alpha}_R\}.$$

4. If the activism elasticity of management is low enough, then investors and society gain from a binding disclosure threshold, but activists do not: $\varepsilon_m^{*R} < \varepsilon_m^* \Rightarrow 0 < \bar{\alpha}_I < \bar{\alpha}_R < \alpha^* \leq \bar{\alpha}_A$.

Figure 1.3 illustrates the results. When the activism elasticity of managerial malfeasance is high, both the regulator and investors want more activism because they gain from deterring malfeasance—neither wants a binding disclosure threshold: $\varepsilon_m^* < \varepsilon_m^{*I}$. In contrast, the activist is harmed by reduced malfeasance and can gain from a threshold that limits his capacity to intervene if management is sensitive enough to the profitability of intervention, i.e., if $\varepsilon_m^* < \varepsilon_m^{*A}$. When, instead, this elasticity is low enough, the ordering of optimal policies is reversed, and the considerations of Proposition 4 dominate for the three parties. Then, investors gain more from a tighter threshold than the regulator ($\varepsilon_m^* > \varepsilon_m^{*I}$) because they incur the trading losses that society does not internalize; the regulator wants a tighter threshold than the activist ($\varepsilon_m^* > \varepsilon_m^{*R}$) because the negative investment feedback harms society; and while lower investment hurts the activist, it does not modify his optimal position (Corollary 5). Investors, activist funds and society can only agree on disclosure thresholds for intermediate activism elasticity levels ε_m , where everyone believes that disclosure thresholds should not bind.

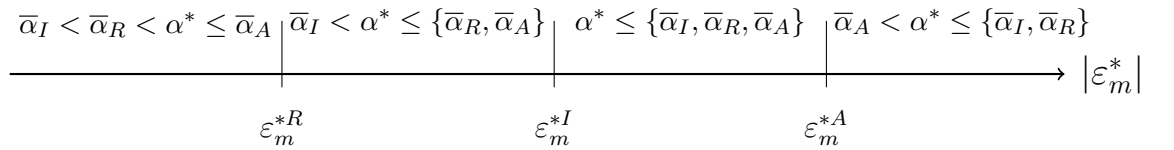


Figure 1.3: Optimal Disclosure Thresholds with Managerial Feedback

The full proof is in the Appendix. Here, we develop the main intuition. Setting $\{\varepsilon_m^{*A}, \varepsilon_m^{*I}, \varepsilon_m^{*R}\} = 0$ and rearranging terms with respect to ε_m^* yields the cutoffs in Proposition 4. Proposition 7 then reveals how those findings are altered when management's behavior is sensitive to the possibility of hedge fund activism.

The activist benefits from a binding disclosure threshold when

$$H(\rho_t)\lambda G(c_t^*) + M_A < 0 \quad (1.16)$$

$$\text{where } M_A \equiv \frac{dH(\rho_t^*)}{dc_t} \lambda G(c_t^*) [c_t^* - E[c|c \leq c_t^*]] < 0.$$

The condition can be rearranged to $\varepsilon_m^* < \varepsilon_m^{*A}$. Here, M_A represents the managerial feedback effect, which hurts the activist—well-behaving management destroys the *raison d'être* of activists. Higher trading profits c_t^* increase the *conditional* profitability of activism, and the extent of activism upon managerial malfeasance $G(c_t^*)$ —Proposition 4. However, it also deters management from acting against uninformed investors, reducing the activist's opportunity to profit. As a result, increasing a binding disclosure threshold, $\bar{\alpha}$, and hence increasing *trading* profits, need not increase the activist's *unconditional* expected profits.

$\frac{dH(\rho_t^*)}{dc_t} = h(\rho_t^*) \frac{d\rho_t^*}{dc_t}$ captures the responsiveness of management to the threat of activism. A large h implies a high activism elasticity of management ε_m , and a large reduction in malfeasance in response to a marginal increase in the conditional profitability of activism. Then, the activist benefits from a disclosure threshold that serves to commit the activist to reducing intervention rates, thereby encouraging managerial malfeasance. In contrast, when h is very small,²⁹ activism does not meaningfully deter managerial malfeasance, and the activism elasticity of management goes to zero. With minimal managerial feedback, $M_A \rightarrow 0$, so (1.16) never holds and predictions reduce to those in Proposition 4: the activist is hurt by a binding disclosure threshold.

The cut-off ε_m^{*A} increases with ε_a^* —the higher is the profit elasticity of activism, the more the activist values a disclosure threshold. When higher trading profits greatly increase the extent of activism, they may also strongly deter managerial malfeasance. Then, the responsiveness ε_a^* of the activist to its potential trad-

²⁹When the second-order conditions hold, local statements about h hold globally for all c_t associated with binding disclosure thresholds.

ing profits harms it—so that the activist gains from a binding disclosure threshold that restrains its responsiveness. In those circumstances, neither investors nor the regulator want a binding disclosure threshold. This reflects that the activist’s gains from a binding disclosure threshold are due to the increased managerial malfeasance that it causes, malfeasance that destroys surplus directly when the activist does not intervene and indirectly when the activist incurs costs of intervention. But then, investors and the regulator value the extensive discouragement effect of potential activism on managerial malfeasance. In particular, when the marginal value to the activist of tightening the disclosure threshold is positive, it is negative for investors and the regulator; and vice versa.

Proposition 7 shows that there exists a range of values $\varepsilon_m^* \in [\varepsilon_m^{*A}, \varepsilon_m^{*I}]$ such that no market participant gains from a binding disclosure threshold. If $\varepsilon_m^* \geq \varepsilon_m^{*A}$, managerial feedback is small enough from the activist’s perspective not to outweigh the benefits of higher conditional profits from participating. Moreover, if $\varepsilon_m^* \leq \varepsilon_m^{*I}$ then the benefits to uninformed investors from deterring managerial malfeasance exceed the associated trading losses of activism, which they incur only if management misbehaves. Thus, investors, too, do not want to limit an activist’s trading profits, even though those profits come at their expense. In particular, investors gain from a binding disclosure threshold if

$$\frac{H(\rho_t^*)\lambda}{f(k)} \left[g(c_t^*) (\delta f(k) - c_t^*) - G(c_t^*) \right] + M_I < 0 \quad (1.17)$$

$$\text{where } M_I \equiv \frac{dH(\rho_t^*)}{dc_t} \frac{\partial \pi_I}{\partial H(\rho_t^*)} > 0,$$

which can be rearranged to $\varepsilon_m^* > \varepsilon_m^{*I}$.

Comparing equations (1.12) and (1.17) reveals the effect of managerial feedback for investors, M_I . Equation (1.12) in Section 3 shows that, absent managerial feedback, investors’ preference over disclosure thresholds only reflects the direct marginal costs and benefits of activism encapsulated in the first term. When

management responds to the threat of activism, the positive effects of activism to investors become twofold: it increases managerial discipline, ex post, *and* it deters managerial malfeasance, ex ante. Thus, managerial feedback reduces the desirability of disclosure thresholds to investors. When $h(\rho)$ is tiny for ρ associated with $c_t \leq c_t^*$, feedback vanishes, so $M_I \rightarrow 0$, and (1.17) reduces to (1.12). When h is higher, management's actions become more sensitive to the extent of activism. As a result, investors may find a binding disclosure threshold undesirable even if the conditional marginal profitability of activism is negative, i.e., even if $g(c_t^*)(\delta f(k) - c_t^*) < G(c_t^*)$. It follows that if investors find a binding disclosure threshold desirable with managerial feedback, then they also do so in the absence of managerial feedback: $\varepsilon_a^* < \varepsilon_a^{*I}$ is necessary for $\varepsilon_m^{*I} < 0$, and hence for (1.17) to hold.

Society does not internalize management's private gains from malfeasance, but is affected by the destruction of project value. A regulator wants a binding disclosure threshold when

$$\left[H(\rho_t^*) \lambda g(c_t^*)(\delta f(k) - c_t^*) + M_R \right] \frac{dc_t^*}{d\alpha} + \pi_A f'(k) \frac{\partial k}{\partial \alpha} < 0 \quad (1.18)$$

where $M_R = M_I f(k) + M_A > 0$,

which can be rearranged to $\varepsilon_m^* > \varepsilon_m^{*R}$. M_R captures the social impact of managerial feedback. Society does not care about transfers of profits between investors and the activist caused by managerial feedback, but only about the aggregate effect, $M_R = M_I f(k) + M_A$. Further expanding M_R reveals that the social benefits of managerial feedback consist of the sum of two elements, weighted by the response of management $\frac{dH(\rho_t)}{dc_t}$ to the threat of activism.³⁰ The regulator wants greater *potential* activism and hence weaker ownership disclosure rules when managers respond by more to the threat of discipline. The first element in the expansion is the value enhanced by deterring malfeasance: $\delta f(k) [1 - \lambda G(c_t)]$. Here, $\delta f(k)$ is the difference

³⁰In particular, $M_R = -\frac{dH(\rho_t)}{dc_t} \left[\delta f(k) [1 - \lambda G(c_t)] + \lambda G(c_t) E[c|c \leq c_t] \right]$.

in firm value under good and bad business plans; and $1 - \lambda G(c_t)$ is the probability that the activist does stop a bad plan when it is implemented. The second element is the expected cost incurred by the activist when it disciplines management, $\lambda G(c_t)E[c|c \leq c_t]$. Deterring malfeasance means that those costs are not incurred.

The sole social cost of activism is a potential reduction in investment. Thus, the regulator must gain from a nonbinding disclosure threshold if it benefits investors: we can only have $\varepsilon_m^{*R} < \varepsilon_m^*$ if investment is reduced by the transfer of trading profits from investors to the activist, i.e., if $\frac{df(k)}{d\alpha} = f'(k)\frac{\partial k}{\partial \alpha} < 0$.³¹ The profit elasticity of activism ε_a^* reduces the harmful effects of negative investment feedback, raising the optimal disclosure threshold.

1.5 Concluding Remarks

Hedge fund activism has generated a regulatory debate about the desirability of revising blockholder disclosure thresholds. These rules were set to protect small investors from abusive tactics of blockholders. We identify the tradeoffs. Disclosure thresholds may discourage activist funds from intervening to protect small investors from corporate managers that can take actions that benefit themselves at the expense of firm value; but activist funds are also informed traders who profit from trading on their information advantage about their value-enhancing actions at the expense of uninformed investors. While managerial discipline creates value and incentivizes real investment, the associated trading rents extracted from uninformed investors reduce their profitability and impair investment, destroying value.

We show that the preferences for binding disclosure thresholds of investors, activist funds and society are never aligned. Whenever activists gain from a binding

³¹To satisfy $\varepsilon_m^{*R} < \varepsilon_m^*$, the cutoff ε_m^{*R} must be negative. The denominator is positive because $-\frac{\partial \pi_I / \partial H(\varphi_t^*)}{\partial \pi_A / \partial H(\varphi_t^*)} > 1$ whereas $\frac{c_t^* - E[c|c \leq c_t^*]}{c_t^*} < 1$. Thus, $\varepsilon_m^{*R} < 0$ if and only if $\frac{\delta f(k) - c_t^*}{c_t^*} + \frac{1}{\varepsilon_a^*} \left(\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t^*/d\alpha}{c_t^*}} \right) < 0$.

threshold—which commits them to intervening less frequently, encouraging managerial misbehavior—investors and society are harmed. Managerial discipline increases investment value without the need for investors to incur further trading losses, and the increased investment benefit society. When, instead, investors gain from a binding threshold, they benefit more than regulators, and activists are necessarily harmed even when the excessive trading losses cause investors to reduce their investments. We only find scope for agreement when all market participants gain from non-binding disclosure thresholds. This requires that the willingness of activists to intervene be sufficiently sensitive to the degree of market opacity, but, in turn, that firm management not be too sensitive to the threat of activism in its choices of whether to take actions that benefit itself at the expense of shareholders.

1.6 Appendix A: Proofs

1.6.1 Proof Proposition 1

Market maker. Let $\hat{\alpha}$ be the market maker's conjecture about the activist trade. Denote $\hat{c}_t \equiv c_t(\hat{\alpha})$ the corresponding conjecture about his cost participation threshold.

The market maker observes net order flow ω . This is either due to (i) the activist did not take a position and $l = -\omega$; (ii) the activist participates and $l = -\omega + \hat{\alpha}$. The activist does not participate when the business plan is good, and when the business plan is bad and either he does not observe it and/or the cost to discipline management is too high. The unconditional probability is $[1 - z\lambda G(\hat{c}_t)]x(-\omega)$. The activist participates if he observes the bad business plan and the cost to discipline management is sufficiently small, which has unconditional probability $z\lambda G(\hat{c}_t)x(-\omega + \hat{\alpha})$. Thus, the expected value of the project is

$$E[V] = \left[\frac{x(-\omega)(1-z) + x(-\omega + \hat{\alpha})z\lambda G(\hat{c}_t)}{x(-\omega)(1-z) + x(-\omega + \hat{\alpha})z\lambda G(\hat{c}_t) + x(-\omega)z[1 - \lambda G(\hat{c}_t)]} \right] f(k) \quad (1.19)$$

$$+ \left[\frac{x(-\omega)z[1 - \lambda G(\hat{c}_t)]}{x(-\omega)(1-z) + x(-\omega + \hat{\alpha})z\lambda G(\hat{c}_t) + x(-\omega)z[1 - \lambda G(\hat{c}_t)]} \right] (1-\delta)f(k)$$

Suppose the market maker observes $\omega < -b + \hat{\alpha}$. Then $x(-\omega + \hat{\alpha}) = 0$, implying that the activist does not participate and $P(\omega) = P_l$. Similarly, suppose that $\omega > 0$. Then $x(-\omega) = 0$, and the activist participates with certainty so $P(\omega) = P_h$. Finally, suppose that some $\omega \in [-b + \hat{\alpha}, 0]$ is observed. Then, the market maker does not know whether the activist participates, and $x(-\omega + \hat{\alpha}) = x(-\omega) = 1/b$, cancelling out of the numerator and denominator. The denominator becomes one and the conditional expected value of the project simplifies to $P(\omega) = P_m$.

Activist. The activist position $\alpha^* = b/2$ is derived in the main text, and the market maker's conjecture is correct in equilibrium, i.e., $\hat{\alpha} = \alpha^*$. Uniqueness of c_t follows from the fact that the left-hand side of (7) increases with c_t , whereas

the right-hand side decreases with c_t . To study c_t as a function of α and k , define $F \triangleq c_t - z[1 - \lambda G(c_t)] \left(\frac{b-\alpha}{b}\right) \alpha \delta f(k)$. From the Implicit Function Theorem, $\frac{\partial c_t}{\partial q} = -\frac{\partial F/\partial q}{\partial F/\partial c_t}$. Thus, we have

$$\begin{aligned}\frac{\partial c_t}{\partial \alpha} &= \frac{z\delta [1 - \lambda G(c_t)] (b - 2\alpha) f(k)}{b + \lambda g(c_t) z\delta (b - \alpha) \alpha f(k)}, \\ \frac{\partial c_t}{\partial k} &= \frac{z\delta [1 - \lambda G(c_t)] (b - \alpha) \alpha f'(k)}{b + \lambda g(c_t) z\delta (b - \alpha) \alpha f(k)}.\end{aligned}\tag{1.20}$$

So $\frac{\partial c_t}{\partial \alpha} > 0 \leftrightarrow \alpha < \alpha^* = \frac{b}{2}$ and $\frac{\partial c_t}{\partial k} > 0$.

1.6.2 Proof of Proposition 2

Gross expected profits. For a full characterization of the gross profit functions at $t = 0$ consider an arbitrary position α . The unconditional project value $E[V]$ in Proposition 2 weighs cash flows $f(k)$ with the probabilities that (i) the manager implements the good business plan, $1 - z$; (ii) the manager implements the bad plan but is disciplined by the activist, $z\lambda G(c_t)$; (iii) the manager implements the bad plan and is not disciplined by the activist but the project succeeds anyway, $z[1 - \lambda G(c_t)](1 - \delta)$.

The activist's gross profits are obtained by weighting his conditional profits $E[\Pi_A|a_1]$ with the probability of participation $z\lambda G(c_t)$;

$$\begin{aligned}E[\Pi_A] &= \pi_A f(k) \\ \text{with } \pi_A &= z\lambda G(c_t) z[1 - \lambda G(c_t)] \left(\frac{b - \alpha}{b}\right) \alpha \delta.\end{aligned}\tag{1.21}$$

By construction, expected investors' profits are the residual $E[\Pi_I] = [\pi_V - \pi_A] f(k)$,

$$\begin{aligned}E[\Pi_I] &= \pi_I f(k) \\ \text{with } \pi_I &= (1 - z) + z\lambda G(c_t) \left[1 - z[1 - \lambda G(c_t)] \left(\frac{b - \alpha}{b}\right) \alpha \delta\right] \\ &\quad + z[1 - \lambda G(c_t)](1 - \delta).\end{aligned}\tag{1.22}$$

Proposition 2 provides expressions for expected profits in equilibrium, incorporating $\alpha = \alpha^* = b/2$. We rearrange π_A as a function of c_t to show that α affects expected profits only through trading transfers c_t and capital, i.e., $E[\Pi_A](c_t(\alpha), k(\alpha))$ and $E[\Pi_I](c_t(\alpha), k(\alpha))$.

Real Investment. The first-order condition for investors' *net* profits $\pi_I f(k) - rk$ characterizes real investment. Note that while π_I is function of both activism and investment, small investors are price takers and do not internalize the effects of their own investment.

1.6.3 Proof of Proposition 4

Investors. We derive the optimal disclosure threshold for investors. Their net expected profits are $\pi_I f(k) - rk$. We differentiate with respect to α to characterize how their marginal profits vary with the activist's position:

$$\frac{d}{d\alpha} \{\pi_I f(k) - rk\} = \left[\frac{\partial \pi_I}{\partial c_t} \frac{dc_t}{d\alpha} + \frac{\partial \pi_I}{\partial k} \frac{\partial k}{\partial \alpha} \right] f(k) + [\pi_I f'(k) - r] \frac{\partial k}{\partial \alpha} \quad (1.23)$$

We show that (1.23) is strictly positive at $\alpha = 0$, implying that investors always benefit from some degree of market opacity, i.e., $\bar{\alpha}_I > 0$. We then prove that (1.23) decreases in α for $\alpha < \alpha^*$. Therefore, if (1.23) is negative at $\alpha = \alpha^*$, then the optimal disclosure threshold $\bar{\alpha}_I$ solves (1.23) = 0 and $\bar{\alpha}_I < \alpha^*$. If, instead, (1.23) is positive at $\alpha = \alpha^*$, then the optimal threshold is non-binding, i.e., $\bar{\alpha}_I \geq \alpha^*$.

Analysis of (1.23) simplifies because of two properties. First, Proposition 2 shows that in equilibrium $\pi_I f'(k) - r = 0$, so the last term of (1.23) vanishes. Second, any interior maximum of $\pi_I f(k) - rk$ satisfies $\frac{\partial k}{\partial \alpha} = 0$ because the activist position that maximizes investor profits, also maximizes investment.³² Using these two features and the expansion $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \alpha}$, it follows from (1.23) that any

³²From the Implicit Function Theorem, $\frac{\partial k}{\partial \alpha} = -\frac{\frac{\partial \pi_I}{\partial c_t} \frac{\partial c_t}{\partial \alpha} f'(k)}{\pi_I f''(k)}$, where the denominator is negative. The proof of Proposition 4 continues by showing that the numerator in this expression characterizes the sign of (1.23). Therefore, $\frac{\partial k}{\partial \alpha} > 0$ if and only if $\frac{d}{d\alpha} \{\pi_I f(k) - rk\} > 0$.

interior solution $\bar{\alpha}_I < \alpha^*$ solves $\frac{\partial \pi_I}{\partial c_t} \frac{\partial c_t}{\partial \alpha} f(k) = 0$.

The proof of Proposition 1 shows that $\frac{\partial c_t}{\partial \alpha} > 0 \leftrightarrow \alpha < \alpha^* = b/2$ with $\frac{\partial c_t}{\partial \alpha} = 0$ for $\alpha < \alpha^*$. Therefore, if there is an interior solution $\bar{\alpha}_I < \alpha^*$, it must be characterized by $\frac{\partial \pi_I}{\partial c_t} = 0$, where

$$\frac{\partial \pi_I}{\partial c_t} = \frac{z\lambda}{f(k)} [g(c_t)(\delta f(k) - c_t) - G(c_t)]. \quad (1.24)$$

Because $g(c)$ is decreasing in c , (1.24) decreases with c_t .

At $\alpha = 0$ activist trading profits are zero, i.e., $c_t = 0$. It follows that if $\alpha = 0$ then (1.24) > 0 , and thus (1.23) > 0 . Therefore, investors always benefit from some degree of market opacity, i.e., from $\bar{\alpha}_I > 0$. We prove below that trading transfers increase with the activist's position $\alpha < \alpha^*$ despite investment feedback, i.e., that $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$ —see the *activist* section of the proof. Hence, (1.24) decreases with α for $\alpha < \alpha^*$, and the same is true for (1.23). A binding optimal threshold exists if and only if (1.23) < 0 for $\alpha = \alpha^*$. Moreover, it satisfies (1.23) $= 0$. The condition (1.23) < 0 can be rearranged as $\varepsilon_a^{*I} < \varepsilon_a^*$.

Activist. We derive the activist's optimal disclosure threshold. His net expected profits are

$$\pi_A f(k) - z\lambda G(c_t) E[c|c \leq c_t] = z\lambda G(c_t) [c_t - E[c|c \leq c_t]], \quad (1.25)$$

where the right-hand side uses the expression for π_A in Proposition 2. Here, $z\lambda G(c_t)$ is the probability that the activist participates, i.e., the probability that (i) the manager adopts the bad business plan, (ii) the activist observes it, and (iii) his cost of intervention is sufficiently small. Conditional on intervention being optimal, his expected profits are the difference between trading profits c_t and the cost of disciplining management, which is expected to be $E[c|c \leq c_t] = [\int_0^{c_t} cg(c) dc] / G(c_t)$.

Differentiating with respect to α yields the marginal profitability to the ac-

tivist of increasing his position:

$$\frac{d}{d\alpha} \{z\lambda G(c_t) [c_t - E[c|c \leq c_t]]\} = z\lambda G(c_t) \frac{dc_t}{d\alpha} \quad (1.26)$$

The result follows because

$$\begin{aligned} \frac{dE[c|c \leq c_t]}{d\alpha} &= \frac{\partial E[c|c \leq c_t]}{\partial c_t} \frac{dc_t}{d\alpha} \\ &= \frac{g(c_t)}{G(c_t)} \left[\frac{\frac{\partial}{\partial c_t} \left\{ \int_0^{c_t} c g(c) dc \right\}}{g(c_t)} - E[c|c \leq c_t] \right] \frac{dc_t}{d\alpha} \\ &= \frac{g(c_t)}{G(c_t)} [c_t - E[c|c \leq c_t]] \frac{dc_t}{d\alpha} \end{aligned} \quad (1.27)$$

where the last line uses $\frac{\partial}{\partial c_t} \left\{ \int_0^{c_t} c g(c) dc \right\} = g(c_t) c_t$.

The sign of (1.26) is determined by $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \alpha}$. The Proof of Proposition 1 shows $\frac{\partial c_t}{\partial \alpha} > 0 \Leftrightarrow \alpha < \alpha^* = \frac{b}{2}$ and also that $\frac{\partial c_t}{\partial k} > 0$. Hence, for the activist to gain from a binding disclosure threshold it must be that $\frac{\partial k}{\partial \alpha} < -\frac{\partial c_t / \partial \alpha}{\partial c_t / \partial k}$ for $\alpha < \alpha^*$, i.e., that the negative investment response to activism by investors is strong enough to outweigh the positive marginal net trading transfers. We prove that this cannot be so by contradiction.

If $\frac{\partial k}{\partial \alpha} < 0$, a marginal increase in the activist's position must hurt investors, implying that $\frac{\partial \pi_I}{\partial c_t} < 0$. Suppose that the investment feedback satisfies $\frac{\partial k}{\partial \alpha} < -\frac{\partial c_t / \partial \alpha}{\partial c_t / \partial k}$ and thus that $\frac{dc_t}{d\alpha} < 0$. By assumption increasing α reduces c_t , so it must increase *investor* profits because $\frac{\partial \pi_I}{\partial c_t} < 0$. But this higher profitability leads investors to increase capital when the activist increases his position $\frac{\partial k}{\partial \alpha} > 0$, a contradiction. It follows that $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$.

This argument establishes that $\bar{\alpha}_A \geq \alpha^*$ in the absence of managerial feedback and yields Corollary 5. This result was also used in the derivation of the optimal disclosure threshold for investors, above.

Regulator. We derive the optimal disclosure threshold for society. The regulator

maximizes the project value net of both cost of capital and expected cost of activism, maximizing

$$\pi_V f(k) - rk - z\lambda G(c_t) E[c|c \leq c_t], \quad (1.28)$$

where π_V is described in Proposition 2. Differentiating (1.28) with respect to α yields the marginal benefit to the regulator of increasing the activist's position:

$$\begin{aligned} & \frac{d}{d\alpha} \{ \pi_V f(k) - rk - z\lambda G(c_t) E[c|c \leq c_t] \} \\ &= \frac{\partial \pi_V}{\partial c_t} \frac{dc_t}{d\alpha} f(k) + \pi_V f'(k) \frac{\partial k}{\partial \alpha} - r \frac{\partial k}{\partial \alpha} - z\lambda \left[g(c_t) E[c|c \leq c_t] + G(c_t) \frac{dE[c|c \leq c_t]}{dc_t} \right] \frac{dc_t}{d\alpha} \end{aligned} \quad (1.29)$$

$$= \frac{\partial \pi_V}{\partial c_t} \frac{dc_t}{d\alpha} f(k) + [\pi_V f'(k) - r] \frac{\partial k}{\partial \alpha} - z\lambda g(c_t) c_t \frac{dc_t}{d\alpha} \quad (1.30)$$

$$= \left[\frac{\partial \pi_V}{\partial c_t} f(k) - z\lambda g(c_t) c_t \right] \frac{dc_t}{d\alpha} + \pi_A f'(k) \frac{\partial k}{\partial \alpha}. \quad (1.31)$$

where the second equality (1.30) uses (1.27); the third equality (1.31) rearranges (1.30) using both equilibrium relationships $\pi_I f'(k) - r = 0$ (optimal investment by the investors) and $\pi_V = \pi_I + \pi_A$ from Proposition 2. Next, in (1.31) substitute for $\frac{\partial \pi_V}{\partial c_t} = z\delta\lambda g(c_t)$ obtained by differentiating the expression for π_V in Proposition 2, and use $\pi_A = z\lambda G(c_t) \frac{c_t}{f(k)}$ to obtain

$$\begin{aligned} (1.29) &= z\lambda g(c_t) [\delta f(k) - c_t] \frac{dc_t}{d\alpha} + z\lambda G(c_t) \frac{c_t}{f(k)} f'(k) \frac{\partial k}{\partial \alpha} \\ &\stackrel{sign}{=} g(c_t) [\delta f(k) - c_t] \frac{dc_t}{d\alpha} + G(c_t) c_t \frac{f'(k)}{f(k)} \frac{\partial k}{\partial \alpha}. \end{aligned} \quad (1.32)$$

The first line of (1.32) corresponds to the condition in (14). The second line of (1.32) can be rearranged to obtain the expression for ε_a^{*R} in Proposition 4 by noting that $\frac{df(k)}{d\alpha} = f'(k) \frac{\partial k}{\partial \alpha}$.

It has been shown that $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$; from (7) it follows that $\delta f(k) - c_t > 0$. Thus, $\frac{\partial k}{\partial \alpha} < 0$ is a necessary condition for a binding disclosure threshold to be optimal for the regulator, and $\varepsilon_a^{*R} < \varepsilon_a^{*I}$.

1.6.4 Proof of Corollary 6

We prove Corollary 6 in three steps.

1. A transfer $\tau > 0$ always reduces $g(c_t)$ and increases $G(c_t)$. A transfer $\tau > 0$ creates both a direct and an indirect effect on $g(c_t)$. The *direct* effect reduces $g(c_t)$ and increases $G(c_t)$ for any given $c_t \in (0, C]$. The *indirect* effect reduces c_t . In particular, from both (7) and the Implicit Function Theorem, c_t decreases in G . Thus, the increase in G caused by the direct effect diminishes c_t . The two effects have opposite effects on $g(c_t)$, but the direct effect always outweighs the indirect effect, so the transfer unambiguously reduces $g(c_t)$ and increases $G(c_t)$. To see this, suppose that a transfer $\tau > 0$ leads to a bigger $g(c_t)$, so the decrease in c_t outweighs the reduction of g . It follows that $G(c_t)$ is smaller, and therefore c_t is larger, a contradiction.

2. There exist cutoffs τ^I and τ^R such that $\varepsilon_a^* < \varepsilon_a^{*I}$ if and only if $\tau^I > \tau$, and $\varepsilon_a^* < \varepsilon_a^{*R}$ if and only if $\tau^R > \tau$. Moreover, $\tau^I < \tau^R$. We showed that any transfer $\tau > 0$ reduces both c_t and $g(c_t)$, and increases $G(c_t)$. From the characterizations provided in Proposition 4, it follows that ε_a^* , ε_a^{*I} and ε_a^{*R} decrease with a transfer $\tau > 0$. Consider now the biggest possible transfer $\tau = \sup \{g\}$, so that all probability mass accumulates at $c = 0$. Then, for any $c_t > 0$, we have $g(c_t) = 0$ and $G(c_t) = 1$. From the characterizations of cutoffs ε_a^* , ε_a^{*I} and ε_a^{*R} in Proposition 4, it follows that a transfer $\tau = \sup \{g\}$ yields $\varepsilon_a^* = 0$, and $\varepsilon_a^{*I} > \varepsilon_a^{*R} > 0$. By continuity, there exist cutoffs $\{\tau^I, \tau^R\} \in (0, \sup \{g\}]$. Since $\varepsilon_a^{*R} < \varepsilon_a^{*I}$, these thresholds satisfy $\tau^I < \tau^R$.

3. Both cutoffs τ^I and τ^R decrease with market liquidity b . Everything else equal, higher liquidity b increases c_t , and decreases the marginal profits of investors $g(c_t) [\delta f(k) - c_t] - G(c_t)$. Thus, a smaller transfer τ^I is required for $\varepsilon_a^{*I} < \varepsilon_a^*$. When $g(c_t) [\delta f(k) - c_t] < G(c_t)$, an increase in trading transfers c_t makes investors' marginal profits more negative, and increases the negative investment feedback

$\frac{\partial k}{\partial \alpha} < 0$. From (1.32) it follows that eventually marginal profits for society become negative.

1.6.5 Proof of Proposition 7

We derive the critical cutoffs $\{\varepsilon_m^I, \varepsilon_m^A, \varepsilon_m^R\}$ in an analysis that mirrors that in the proof of Proposition 4 incorporating $z \equiv H\left(\varphi\left[\frac{1-\lambda G(c_t)}{\lambda G(c_t)}\right]\right)$. The proof continues to compare the cutoffs and derive the implications for the optimal disclosure thresholds of market participants. Finally it shows that second order conditions hold for uniformly distributed cost of activism and reputation cost.

As a preliminary step, we verify that the partial effects of α and k on the trading transfers c_t preserve the same sign. From the Implicit Function Theorem,

$$\begin{aligned}\frac{\partial c_t}{\partial \alpha} &= \frac{H(\rho_t) [1 - \lambda G(c_t)] (b - 2\alpha) \delta f(k)}{b + \left[H(\rho_t) \lambda g(c_t) - h(\rho_t) \frac{d\rho_t}{dc_t} [1 - \lambda G(c_t)] \right] (b - \alpha) \alpha \delta f(k)} \\ \frac{\partial c_t}{\partial k} &= \frac{H(\rho_t) \delta [1 - \lambda G(c_t)] (b - \alpha) \alpha f'(k)}{b + \left[H(\rho_t) \lambda g(c_t) - h(\rho_t) \frac{d\rho_t}{dc_t} [1 - \lambda G(c_t)] \right] (b - \alpha) \alpha \delta f(k)}\end{aligned}\quad (1.33)$$

where $\frac{d\rho_t}{dc_t} = -\frac{\varphi}{\lambda} \frac{g(c_t)}{G(c_t)^2}$. Therefore, $\frac{\partial c_t}{\partial \alpha} > 0 \leftrightarrow \alpha < \alpha^* = \frac{b}{2}$ and $\frac{\partial c_t}{\partial k} > 0$.

Investors. The derivative of investors' net profits with respect to α is given by (1.23). If no interior solution exists, then investors do not benefit from a disclosure threshold, i.e., $\bar{\alpha}_I \geq \alpha^*$.

In equilibrium, $\pi_I f'(k) - r = 0$ and $\frac{\partial k}{\partial \alpha} = 0$ at an interior maximum, $\bar{\alpha}_I < \alpha^*$. Use $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \alpha}$ to simplify (1.23) to $\frac{\partial \pi_I}{\partial c_t} \frac{\partial c_t}{\partial \alpha} f(k)$. We have $\frac{\partial c_t}{\partial \alpha} > 0 \leftrightarrow \alpha < \alpha^* = b/2$ and $\frac{\partial c_t}{\partial k} > 0$. Hence, an interior maximum $\bar{\alpha}_I < \alpha^*$ is characterized by $\frac{\partial \pi_I}{\partial c_t} = 0$, where

$$\frac{\partial \pi_I}{\partial c_t} = \frac{H(\rho_t) \lambda}{f(k)} [g(c_t) (\delta f(k) - c_t) - G(c_t)] + \underbrace{\frac{dH(\rho_t)}{dc_t} \frac{\partial \pi_I}{\partial H(\rho_t)}}_{M_I} \quad (1.34)$$

Here, $\frac{dH(\rho_t)}{dc_t} = h(\rho_t) \frac{d\rho_t}{dc_t} = g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)}$ and $\frac{\partial \pi_I}{\partial H(\rho_t)} = - \left[\delta(1 - \lambda G(c_t)) + \lambda G(c_t) \frac{c_t}{f(k)} \right]$.

Because $M_I > 0$, managerial feedback raises the marginal profitability of a higher cutoff to investors.

At $\alpha = 0$, activist trading profits are zero, so $c_t = 0$, and hence (1.34) > 0 and thus (1.23) > 0 : investors always benefit from some degree of market opacity, i.e., $\bar{\alpha}_I > 0$. To characterize ε_m^I , rearrange (1.34) = 0 as:

$$0 = \frac{H(\rho_t) \lambda}{f(k)} [g(c_t) (\delta f(k) - c_t) - G(c_t)] + g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{\partial \pi_I}{\partial H(\rho_t)}. \quad (1.35)$$

Substituting $\varepsilon_a = \frac{g(c_t)}{G(c_t)} c_t$ and $\varepsilon_m = \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{G(c_t)}{H(\rho_t)}$ into (1.35), we next divide by $H(\rho_t)$ and successively rearrange to obtain:

$$\begin{aligned} 0 &= \frac{\lambda}{f(k)} \left[\varepsilon_a \left(\frac{\delta f(k) - c_t}{c_t} \right) - 1 \right] + \varepsilon_m \frac{g(c_t)}{G(c_t)^2} \frac{\partial \pi_I}{\partial H(\rho_t)}; \\ &= \frac{\lambda}{f(k)} \left[\varepsilon_a \left(\frac{\delta f(k) - c_t}{c_t} \right) - 1 \right] + \varepsilon_m \varepsilon_a \frac{1}{G(c_t) c_t} \frac{\partial \pi_I}{\partial H(\rho_t)}; \\ &\Rightarrow - \left(\frac{\lambda G(c_t) \frac{c_t}{f(k)}}{\partial \pi_I / \partial H(\rho_t)} \right) \left[\frac{\delta f(k) - c_t}{c_t} - \frac{1}{\varepsilon_a} \right] = \varepsilon_m. \end{aligned} \quad (1.36)$$

The expression for ε_m^I then follows directly from $\lambda G(c_t) \frac{c_t}{f(k)} = \frac{\partial \pi_A}{\partial H(\rho_t)}$. When investors value a binding disclosure threshold, i.e., when $\bar{\alpha}_I < \alpha^*$, it satisfies $\varepsilon_m = \varepsilon_m^I$.

Activist. The activist's expected net profits are given by (1.25). Differentiating with respect to α yields

$$\begin{aligned} &\frac{d}{d\alpha} \{H(\rho_t) \lambda G(c_t) [c_t - E[c|c \leq c_t]]\} \\ &= \left[H(\rho_t) \lambda G(c_t) + \underbrace{\frac{dH(\rho_t)}{dc_t} \lambda G(c_t) [c_t - E[c|c \leq c_t]]}_{M_A} \right] \frac{dc_t}{d\alpha}. \end{aligned} \quad (1.37)$$

where $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \alpha}$ was derived in the proof of Proposition 4. Because $M_A < 0$, managerial feedback reduces the marginal profits from increasing α to the activist.

Since (1.37) > 0 at $\alpha = 0$, the activist always benefits from some degree of market opacity, i.e., $\bar{\alpha}_A > 0$. If no interior solution exists, then the activist does not benefit from a disclosure threshold, i.e., $\bar{\alpha}_A \geq \alpha^*$. Set (1.37) $= 0$ to derive ε_m^A , and recall that $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$. Thus, (1.37) $= 0$ implies

$$H(\rho_t) + \frac{dH(\rho_t)}{dc_t} [c_t - E[c|c \leq c_t]] = 0. \quad (1.38)$$

Substitute $\frac{dH(\rho_t)}{dc_t} = g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)}$ and the expressions for elasticities $\varepsilon_a = \frac{g(c_t)}{G(c_t)} c_t$ and $\varepsilon_m = \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{G(c_t)}{H(\rho_t)}$ into (1.38) and divide by $H(\rho_t)$ to obtain:

$$\begin{aligned} 0 &= 1 + g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{1}{H(\rho_t)} [c_t - E[c|c \leq c_t]] \\ &= 1 + \frac{g(c_t)}{G(c_t)} \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{G(c_t)}{H(\rho_t)} [c_t - E[c|c \leq c_t]] \\ &= 1 + \varepsilon_m \varepsilon_a \left[\frac{c_t - E[c|c \leq c_t]}{c_t} \right]. \end{aligned} \quad (1.39)$$

The characterization of ε_m^A follows directly.

Regulator. The regulator's net expected payoff is given by (1.28). Differentiating with respect to α yields the marginal payoff to the regulator of increasing the activist's position:

$$\begin{aligned} &\frac{d\pi_V}{dc_t} \frac{dc_t}{d\alpha} f(k) + \pi_V f'(k) \frac{\partial k}{\partial \alpha} - r \frac{\partial k}{\partial \alpha} - \frac{dH(\rho_t)}{dc_t} \lambda G(c_t) E[c|c \leq c_t] \frac{dc_t}{d\alpha} \\ &- H(\rho_t) \lambda g(c_t) E[c|c \leq c_t] \frac{dc_t}{d\alpha} - H(\rho_t) \lambda G(c_t) \frac{g(c_t)}{G(c_t)} [c_t - E[c|c \leq c_t]] \frac{dc_t}{d\alpha}. \end{aligned} \quad (1.40)$$

Substitute the equilibrium relationship $\pi_I f'(k) - r = 0$ and $\pi_V = \pi_I + \pi_A$ to

rearrange the regulator's marginal payoff from increasing α as:

$$\begin{aligned}
(1.40) &= \frac{d\pi_V}{dc_t} \frac{dc_t}{d\alpha} f(k) + \pi_A f'(k) \frac{\partial k}{\partial \alpha} \\
&\quad - \frac{dH(\rho_t)}{dc_t} \lambda G(c_t) E[c|c \leq c_t] \frac{dc_t}{d\alpha} - H(\rho_t) \lambda g(c_t) c_t \frac{dc_t}{d\alpha} \\
&= -\frac{dH(\rho_t)}{dc_t} \left[\delta f(k) [1 - \lambda G(c_t)] + \lambda G(c_t) E[c|c \leq c_t] \right] \frac{dc_t}{d\alpha} \\
&\quad + H(\rho_t) \lambda g(c_t) [\delta f(k) - c_t] \frac{dc_t}{d\alpha} + \pi_A f'(k) \frac{\partial k}{\partial \alpha}
\end{aligned} \tag{1.41}$$

where the second equality follows from $\frac{d\pi_V}{dc_t} = H(\rho_t) \delta \lambda g(c_t) - \frac{dH(\rho_t)}{dc_t} \delta [1 - \lambda G(c_t)]$.

Rearranging yet again yields:

$$\begin{aligned}
(1.40) &= \left[H(\rho_t) \lambda g(c_t) [\delta f(k) - c_t] + M_R \right] \frac{dc_t}{d\alpha} + \pi_A f'(k) \frac{\partial k}{\partial \alpha}, \tag{1.42} \\
\text{where } M_R &\equiv -\frac{dH(\rho_t)}{dc_t} \left[\delta f(k) [1 - \lambda G(c_t)] + \lambda G(c_t) E[c|c \leq c_t] \right].
\end{aligned}$$

This equation corresponds to the characterization in (1.18). Since $M_R > 0$, managerial feedback increases the marginal profitability to the regulator of increasing the activist's position.

To ease exposition, we define $\Psi \equiv -\frac{\partial \pi_V}{\partial H(\rho_t)} f(k) + \lambda G(c_t) E[c|c \leq c_t]$ so that $M_R = -\frac{dH(\rho_t)}{dc_t} \Psi$. Moreover, recall that $\frac{dH(\rho_t)}{dc_t} = g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)}$. Substituting, rewrite (1.42) as:

$$\begin{aligned}
0 &= \left[-\frac{\partial H(\rho_t)}{\partial G(c_t)} g(c_t) \Psi + H(\rho_t) \lambda g(c_t) (\delta f(k) - c_t) \right] \frac{dc_t}{d\alpha} + \pi_A \frac{df(k)}{d\alpha} \tag{1.43} \\
&= g(c_t) (\delta f(k) - c_t) - \frac{g(c_t)}{G(c_t)} \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{G(c_t)}{H(\rho_t)} \frac{\Psi}{\lambda} + \frac{\pi_A}{H(\rho_t) \lambda} \frac{df(k)/d\alpha}{dc_t/d\alpha} \\
&= \varepsilon_a \left(\frac{\delta f(k) - c_t}{c_t} \right) G(c_t) - \varepsilon_m \varepsilon_a \frac{\Psi}{\lambda c_t} + \frac{\pi_A}{H(\rho_t) \lambda} \frac{df(k)/d\alpha}{dc_t/d\alpha} \\
&= \varepsilon_a \left(\frac{\delta f(k) - c_t}{c_t} \right) - \varepsilon_m \varepsilon_a \frac{\Psi}{\lambda G(c_t) c_t} + \frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t/d\alpha}{c_t}} \\
&= -\varepsilon_m + \frac{\lambda G(c_t) c_t}{\Psi} \left[\frac{\delta f(k) - c_t}{c_t} + \frac{1}{\varepsilon_a} \left(\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t/d\alpha}{c_t}} \right) \right],
\end{aligned}$$

where the third line uses $\varepsilon_a = \frac{g(c_t)}{G(c_t)} c_t$ and $\varepsilon_m = \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{G(c_t)}{H(\rho_t)}$. To derive ε_m^R note in the last line of (1.43) that using $\pi_V = \pi_I + \pi_A$ we obtain

$$\begin{aligned} \frac{\lambda G(c_t) c_t}{\Psi} &= \left[-\frac{\frac{\partial \pi_I}{\partial H(\rho_t)} f(k)}{\lambda G(c_t) c_t} - \frac{\frac{\partial \pi_A}{\partial H(\rho_t)} f(k)}{\lambda G(c_t) c_t} + \frac{E[c|c \leq c_t]}{c_t} \right]^{-1} \\ &= \left[-\frac{\partial \pi_I / \partial H(\rho_t)}{\partial \pi_A / \partial H(\rho_t)} - \frac{c_t - E[c|c \leq c_t]}{c_t} \right]^{-1}. \end{aligned} \quad (1.44)$$

Cutoff relation. The analysis above rearranges the marginal payoffs to market participants of increasing α . When second order conditions hold, (i) activist marginal profits are decreasing at α^* when $\varepsilon_m^* < \varepsilon_m^{A*}$; (ii) investors' marginal profits are decreasing at α^* if $\varepsilon_m^{I*} < \varepsilon_m^*$; (iii) the regulator's marginal payoff is decreasing at α^* when $\varepsilon_m^{R*} < \varepsilon_m^*$.

Next we show that $\varepsilon_m^A < \varepsilon_m^I < \varepsilon_m^R$. Since marginal profits of all market agents are positive at $\alpha = 0$, $\varepsilon_m \in (\varepsilon_m^A, \varepsilon_m^I)$ when $\alpha = 0$. It follows that if second order conditions hold, when investors want a binding disclosure threshold, activists do not and vice versa. Moreover, and no party wants a binding disclosure threshold if $\varepsilon_m^* \in [\varepsilon_m^{A*}, \varepsilon_m^{I*}]$.

To see that $\varepsilon_m^A < \varepsilon_m^I$, note that the relation is equivalent to

$$\frac{1}{\varepsilon_a} \left(-\frac{\partial \pi_A / \partial H(\rho_t)}{\partial \pi_I / \partial H(\rho_t)} - \frac{c_t}{c_t - E[c|c \leq c_t]} \right) < -\left(\frac{\partial \pi_A / \partial H(\rho_t)}{\partial \pi_I / \partial H(\rho_t)} \right) \left[\frac{\delta f(k) - c_t}{c_t} \right]. \quad (1.45)$$

The left-hand side of (1.45) is negative because

$$-\frac{\partial \pi_A / \partial H(\rho_t)}{\partial \pi_I / \partial H(\rho_t)} = -\frac{\lambda G(c_t) \frac{c_t}{f(k)}}{[1 - \lambda G(c_t)]\delta + \lambda G(c_t) \frac{c_t}{f(k)}} \in (0, 1)$$

whereas $\frac{c_t}{c_t - E[c|c \leq c_t]} > 1$. The right-hand side of (1.45) is positive because $\partial \pi_A / \partial H(\rho_t) > 0$ whereas $\partial \pi_I / \partial H(\rho_t) < 0$, so we have $\varepsilon_m^A < \varepsilon_m^I$.

To see that $\varepsilon_m^I < \varepsilon_m^R$, note that a necessary condition for $\varepsilon_m^R < 0$ is that investment decrease with α , i.e., $\frac{df(k)}{d\alpha} = f'(k)\frac{\partial k}{\partial \alpha} < 0$, which implies that activist marginal profits decrease and thus $\varepsilon_m^I < \varepsilon_m$. Hence, if $\varepsilon_m^R = 0$, then $\varepsilon_m^I < \varepsilon_m < 0$. That is, the sole cost to society of increasing trading transfers is a reduction in real investment, while the benefits exceed those for investors. Thus, for $\varepsilon_m^R < \varepsilon_m$, it is necessary, but not sufficient, that $\varepsilon_m^I < \varepsilon_m$, which implies $\varepsilon_m^R < \varepsilon_m^I$.

The uniform-uniform case

We show that when both c and ρ are uniformly distributed, second-order conditions hold.

Investors. We rewrite the first-order condition for investors in (35), first substituting in the uniform distribution of the manager's cost of reputation, and then the uniform distribution of the activist's cost of intervention. Substituting $H(\rho_t) = \frac{\rho_t}{R}$ and $h(\rho_t) = \frac{1}{R}$, (35) becomes:

$$\begin{aligned} 0 = & \underbrace{\frac{\varphi}{R} \left[\frac{1 - \lambda G(c_t)}{\lambda G(c_t)} \right]}_{H(\rho_t)} \frac{\lambda}{f(k)} [g(c_t)(\delta f(k) - c_t) - G(c_t)] \\ & + \underbrace{\frac{\varphi}{R} \left[\frac{g(c_t)}{\lambda G(c_t)^2} \right]}_{-\partial H(\rho_t)/\partial c_t} \underbrace{\left[\delta(1 - \lambda G(c_t)) + \lambda G(c_t) \frac{c_t}{f(k)} \right]}_{-\partial \pi_I / \partial H(\rho_t)}. \end{aligned} \quad (1.46)$$

Multiplying (1.46) by $\frac{R}{\varphi} \left[\frac{\lambda G(c_t)}{1 - \lambda G(c_t)} \right] \frac{f(k)}{\lambda}$ yields an equivalent condition

$$0 = g(c_t)(\delta f(k) - c_t) - G(c_t) + \left[\frac{g(c_t)}{\lambda G(c_t)} \right] \left[\delta f(k) + \frac{\lambda G(c_t)}{1 - \lambda G(c_t)} c_t \right], \quad (1.47)$$

which we multiply yet again by $\frac{1}{g(c_t)}$ and rearrange to obtain

$$0 = \delta f(k) \left[\frac{1 + \lambda G(c_t)}{\lambda G(c_t)} \right] + c_t \left[\frac{\lambda G(c_t)}{1 - \lambda G(c_t)} \right] - \frac{G(c_t)}{g(c_t)}. \quad (1.48)$$

Substituting $G(c_t) = \frac{c_t}{C}$ and $g(c_t) = \frac{1}{C}$, the first-order condition (1.48) for investors

becomes

$$0 = \delta f(k) \left[\frac{C + \lambda c_t}{\lambda c_t} \right] + c_t \left[\frac{\lambda c_t}{C - \lambda c_t} \right] - c_t \quad (1.49)$$

We prove that there is a unique solution to the first-order condition for investors by showing that the right-hand side (RHS) of (1.49) decreases in c_t . Differentiating yields

$$\frac{d}{dc_t} RHS(1.49) = \left(\frac{C}{C - \lambda c_t} \right)^2 - \frac{\delta f(k) C}{\lambda c_t^2} - 2, \quad (1.50)$$

which is negative for $c_t \rightarrow 0$ and increasing in c_t . We derive an upper bound for c_t and show that $\frac{d}{dc_t} RHS(1.49) < 0$ for such trading transfers, establishing that the solution to the first-order condition is unique. Trading profits c_t are maximized by the optimal conditional trade $\alpha^* = b/2$ with the highest liquidity shock $b = 1$. Substituting into the expression for c_t yields an implicit upper bound on c_t :

$$\begin{aligned} c_t &\leq H(\rho_t) [1 - \lambda G(c_t)] \frac{\delta f(k)}{4} \\ &= \frac{\varphi}{R} \left[\frac{C - \lambda c_t}{\lambda c_t} \right] \left[\frac{C - \lambda c_t}{C} \right] \frac{\delta f(k)}{4}, \end{aligned} \quad (1.51)$$

or equivalently,

$$\left(\frac{C}{C - \lambda c_t} \right)^2 = \frac{\varphi}{R} \frac{C}{\lambda c_t^2} \frac{\delta f(k)}{4}.$$

Plugging the last expression in (1.50) reveals that $\frac{d}{dc_t} RHS(1.49) < 0$ for $\varphi < 4R$.

Activist. Substitute $H(\rho_t) = \frac{\rho_t}{R}$ and $h(\rho_t) = \frac{1}{R}$ to rewrite the activist's first-order condition (38) as:

$$0 = \underbrace{\frac{\varphi}{R} \left[\frac{1 - \lambda G(c_t)}{\lambda G(c_t)} \right]}_{H(\rho_t)} + \underbrace{\left(-\frac{\varphi}{R} \frac{g(c_t)}{\lambda G(c_t)^2} \right)}_{\partial H(\rho_t)/\partial c_t} [c_t - E[c|c \leq c_t]] \quad (1.52)$$

Multiplying (1.52) by $\frac{R}{\varphi} \left[\frac{\lambda G(c_t)}{1 - \lambda G(c_t)} \right]$ yields a simpler, equivalent condition

$$0 = 1 - \left[\frac{g(c_t)}{G(c_t)} \right] \left(\frac{c_t - E[c|c \leq c_t]}{1 - \lambda G(c_t)} \right). \quad (1.53)$$

Substitute $G(c_t) = \frac{c_t}{C}$ and $g(c_t) = \frac{1}{C}$ and note that $c_t - E[c|c \leq c_t] = \frac{c_t}{2}$. It follows that the activist's first-order condition satisfies

$$0 = 1 - \frac{1}{2} \left(\frac{C}{C - \lambda c_t} \right). \quad (1.54)$$

The right-hand side decreases in c_t , implying a unique solution.

1.7 Appendix B: Robustness

1.7.1 Allowing for pure informed trading

We relax the assumption that the activist can only take a position when the manager implements the bad business plan and show that it does not alter our results qualitatively. We reproduce the analysis of the market in Section 2.1 assuming that at $t = 2$, if the activist observes the company, which occurs with probability λ , he can take a position, regardless of the business plan implemented by the manager at $t = 1$.

Proposition 8 *At $t = 2$, if the activist either (a) observes managerial malfeasance ($m = 0$) and the cost of activism is sufficiently small, $c \leq c_t^*$ where*

$$c_t^* = z [1 - \lambda G(c_t^*)] \frac{b}{4} \delta f(k); \quad (1.55)$$

or (b) observes that the manager behaves ($m = 1$); he takes a position

$$\alpha^* = \frac{b}{2} \quad (1.56)$$

and engages in managerial disciplining in situation (a).

The market maker, upon observing the net order flow, sets prices

$$\begin{aligned} P(\omega) = P_l &\equiv \left[\frac{(1-z)(1-\lambda) + z[1 - \lambda G(c_t^*)](1-\delta)}{(1-\lambda) + \lambda z(1 - G(c_t^*))} \right] f(k) && \text{if } \omega < -b + \alpha^* \\ P(\omega) = P_m &\equiv [1 - z(1 - \lambda G(c_t^*))\delta] f(k) && \text{if } \omega \in [-b + \alpha^*, 0] \\ P(\omega) = P_h &\equiv f(k) && \text{if } \omega > 0 \end{aligned} \quad (1.57)$$

We provide the full proof at the end of this section; here we discuss the differences with the model in the main text. Case (a) is equivalent to the setting studied in Section 2.1 and results are equal. The activist intervenes to discipline management if the conditional trading profits of doing so (weakly) outweigh the cost

of intervention, i.e., if $c \leq c_t^*$. Trading profits equal those of the benchmark setting, and the corresponding cost cutoff (1.55) is the same. The position that maximizes trading profits is not altered by the new assumption $\alpha^* = b/2$.

Case (b) captures the difference with respect to the original model, and highlights that the activist can acquire stock when the manager behaves ($m = 1$). It reveals an intuitive result:

Lemma 9 *Pure informed trading occurs if and only if the activist observes that the manager implemented the good business plan. Therefore, it has unconditional probability $(1 - z)\lambda$.*

When the activist observes the good plan, he can profit from his information advantage (trading profits) without the need of incurring any cost. Thus, he always takes a position. Moreover, the activist would never act as a mere informed trader after observing the bad plan. This would imply acquiring overvalued stock, and has negative expected profits.

Notably, when the activist acts as a mere informed trader, he takes the same position $\alpha^* = b/2$. Whether he intends to discipline management or not, a position of $b/2$ maximizes trading profits. If management misbehaves, the activist only participates if these trading profits outweigh the cost of intervention. If management behaves, he always participates (upon observing management's action).

A positive net order flow $\omega > 0$ reveals the activist, and activist participation is associated to certain cash flows $f(k)$ —as in the benchmark model. This is because additional participation only occurs if the business plan is good —case (b). An intermediate order flow $\omega \in [-b + \alpha^*, 0]$ does not provide any information about activist participation, and the conditional value of the project equals the unconditional value. For a given k , the new assumption does not change the value of the project, so the expression for P_m is unchanged from Proposition 1. Price P_l is lower than in the benchmark model. This is because when the absence of the

activist is revealed, i.e., if $\omega < -b + \alpha^*$, it is more likely that the bad business plan was implemented. This is because if the activist had observed a good plan, he would have taken a position.

Proposition 10 *The expected value at $t = 0$ of the project given investment k is*

$$E[V] = [1 - z(1 - \lambda G(c_t^*))\delta]f(k) \equiv \pi_V f(k). \quad (1.58)$$

Expected gross profits of the activist are:

$$E[\Pi_A] = [(1 - z)\lambda + z\lambda G(c_t^*)] \frac{c_t^*}{f(k)} f(k) \equiv \pi_A f(k). \quad (1.59)$$

Expected gross profits of uninformed investors are:

$$E[\Pi_I] = (\pi_V - \pi_A)f(k) \equiv \pi_I f(k). \quad (1.60)$$

Investment by uninformed investors k solves

$$\pi_I f'(k) - r = 0. \quad (1.61)$$

The proof follows directly from that of Proposition 2 in the main text and the following discussion. Given initial investment, the project has the same value $E[V]$ than in the benchmark setting - Proposition 2. The new assumption alters the distribution of revenues between investors and the activist. Equation (1.59) reveals that the activist obtains the same trading profits c_t^* but with higher probability. In particular, trading transfers are realized if either (a) the activist disciplines management, which occurs with probability $z\lambda G(c_t^*)$; (b) the activist acts as a mere informed trader, which has probability $(1 - z)\lambda$. It follows that π_I is smaller than in Proposition 2 and therefore investment levels captured by (1.61) are lower too.

Proof of Proposition 9

Market maker. The market maker's conjecture about activist position is $\hat{\alpha}$, and $\hat{c}_t \equiv c_t(\hat{\alpha})$ the corresponding conjecture about the trading transfers.

The net order flow ω either (i) equals liquidity trade because the activist did not take a position, i.e., $l = -\omega$; or (ii) is the difference between liquidity sales and the activist position, i.e., $l = -\omega + \hat{\alpha}$. The activist does not participate when either he does not observe the company, or he observes that the bad business plan is implemented but it is too costly to intervene. The unconditional probability is $[(1 - \lambda) + z\lambda(1 - G(\hat{c}_t))]x(-\omega)$. The activist participates when he observes the company and either the manager implemented the good business plan, or she implemented the bad business plan but the cost of intervention is sufficiently small. The unconditional probability of taking a position is $[(1 - z)\lambda + z\lambda G(\hat{c}_t)]x(-\omega + \hat{\alpha})$. It follows that the expected value of the project is

$$E[V] = \left[\frac{x(-\omega)(1 - z)(1 - \lambda) + x(-\omega + \hat{\alpha})(1 - z)\lambda + x(-\omega + \hat{\alpha})z\lambda G(\hat{c}_t)}{\left[\begin{array}{c} x(-\omega)(1 - z)(1 - \lambda) + x(-\omega + \hat{\alpha})(1 - z)\lambda + x(-\omega + \hat{\alpha})z\lambda G(\hat{c}_t) \\ + x(-\omega)z(1 - \lambda) + x(-\omega)z\lambda(1 - G(\hat{c}_t)) \end{array} \right]} \right] f(k) \quad (1.62)$$

$$+ \left[\frac{x(-\omega)z(1 - \lambda) + x(-\omega)z\lambda(1 - G(\hat{c}_t))}{\left[\begin{array}{c} x(-\omega)(1 - z)(1 - \lambda) + x(-\omega + \hat{\alpha})(1 - z)\lambda + x(-\omega + \hat{\alpha})z\lambda G(\hat{c}_t) \\ + x(-\omega)z(1 - \lambda) + x(-\omega)z\lambda(1 - G(\hat{c}_t)) \end{array} \right]} \right] (1 - \delta)f(k)$$

Consider the case where the market maker observes $\omega < -b + \hat{\alpha}$. Then $x(-\omega + \hat{\alpha}) = 0$ and $x(-\omega) = 1/b$, implying that the activist does not participate and $P(\omega) = P_l$ if $\hat{\alpha} = \alpha^*$. Similarly, suppose that $\omega > 0$. Then $x(-\omega) = 0$ and $x(-\omega + \hat{\alpha}) = 1/b$, and the activist participates with certainty so $P(\omega) = P_h$. Finally,

consider the case where $\omega \in [-b + \hat{\alpha}, 0]$. Then $x(-\omega) = x(-\omega + \hat{\alpha}) = 1/b$ and the market maker does not know whether the activist participates. The conditional expected value of the project is $P(\omega) = P_m$.

Activist position. When the activist takes a position α , with probability $\int_{-b+\alpha}^0 \frac{1}{b} dl = \frac{b-\alpha}{b}$ the net order flow satisfies $\omega \in [-b + \alpha, 0]$ and he obtains gross profit $f(k) - P_m > 0$ for each share he ordered. With the remaining probability $\frac{\alpha}{b}$, the net order flow is $\omega > 0$ and he pays $P_h = f(k)$, thereby making zero profits. His gross expected profit conditional on participating reads

$$E[\Pi_A|a_{11}] = E[\Pi_A|a_{10}] = \left(\frac{b-\alpha}{b}\right) \alpha [f(k) - P_m] \quad (1.63)$$

and it is maximized for a position $\alpha^* = b/2$.

Consider now an arbitrary $\alpha \leq \alpha^*$. Upon observing $m = 0$, the activist participates if and only if $c \leq E[\Pi_A|a_1]$, implying that the *intervention* cut-off satisfies $c_t = E[\Pi_A|a_1]$. Plugging P_m into $E[\Pi_A|a_1]$ yields

$$c_t = z [1 - \lambda G(c_t)] \left(\frac{b-\alpha}{b}\right) \alpha \delta f(k). \quad (1.64)$$

If, instead, the activist observes $m = 1$, he acts as an informed trader and does not incur any cost. Therefore, he takes a position $\alpha = \alpha^*$ regardless of the realization of c .

1.7.2 Allowing for cut-and-run

We relax the assumption that the activist must incur the cost to discipline management whenever he takes a position in the company and show that it does not affect our results qualitatively. After acquiring stock, the activist discloses his position and this leads to a price increase that makes his intervention profitable. We introduce a new liquidity shock that allows the activist to abandon his position by selling

stock without incurring the cost of intervention, i.e., to cut-and-run. We assume that cut-and-run has a fixed cost of reputation and this is privately known to the activist.

We enrich the original setting so that $t = 2$ has four subperiods, from $t = 2.1$ to $t = 2.4$. At $t = 2.1$, the activist observes the business plan that the manager implemented with probability λ . Upon observing the business plan, the activist can take a position $\alpha \in [0, 1]$, regardless of whether the plan is good or bad (i.e., we allow for pure informed trading) in a dealership market where there is also liquidity trade of $l_1 \sim U[-b, b]$. At $t = 2.2$, the activist can disclose his position if it was not previously disclosed by his stock purchase. At $t = 2.3$, if the activist has a position in the company, he can exit by selling shares $\mu \leq [0, \alpha]$ in a dealership market that also has liquidity trade of $l_3 \sim U[-b, b]$. At $t = 2.4$, the activist decides whether to discipline management and incur the cost c , which is privately observed. Unwinding a position without trying to discipline management after disclosure, i.e., cutting and running, has a privately known reputation cost ς . Other agents have the prior that ς is distributed on $[0, \Sigma]$ according to a strictly positive density s and associated cumulative function S . We assume that s is weakly decreasing and satisfies the monotone hazard ratio property. Figure 1.4 details the sequence of events.

We allow liquidity shocks to be positive or negative to provide the activist an opportunity to camouflage stock sales after revealing his position, preserving the key role of liquidity for informed trading. For simplicity we assume that the distributions of liquidity trade at $t = 2.1$ and $t = 2.3$ to be the same. In reality, the activist likely has a better opportunity to conceal his trade when he takes a position ($t = 2.1$) than when he abandons his position after disclosure ($t = 2.3$). This is because other market participants may pay closer attention when they know that the activist has a position, making cutting-and-running less profitable. Our setting can be viewed as providing an upper bound on the profits of a cut-and-run

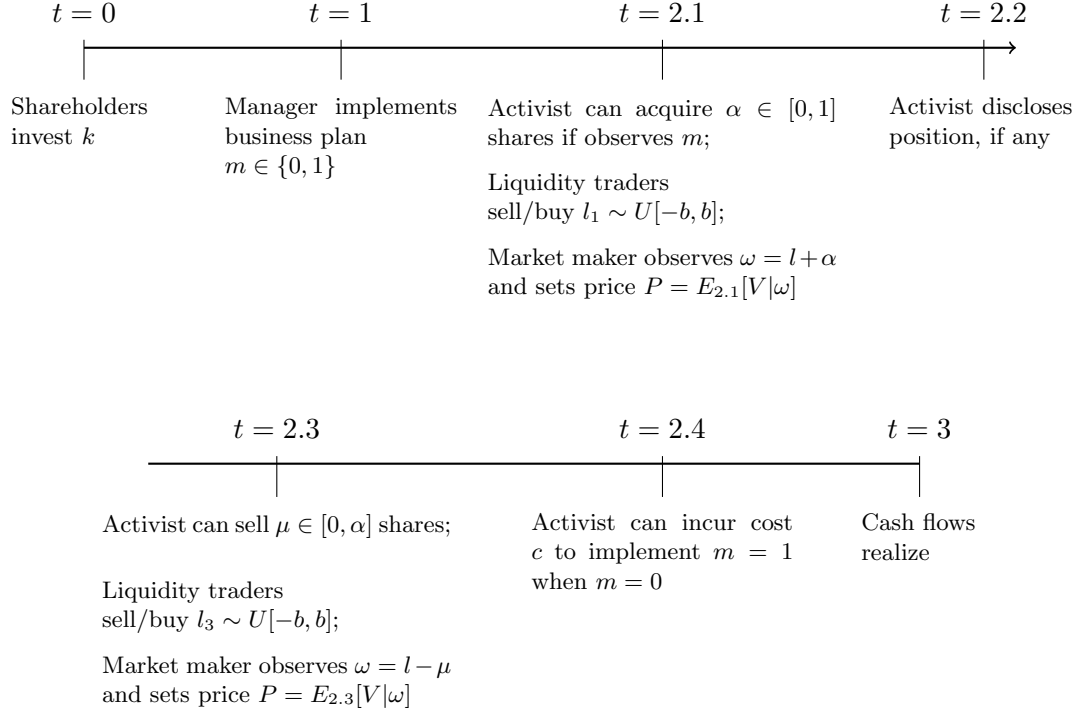


Figure 1.4: Time line with cut-and-run

strategy.³³ Despite this, we show that a cut-and-run strategy is never profitable in our benchmark setting.

The assumption that cutting-and-running damages reputations is consistent both with anecdotes and with more systematic evidence by [Johnson and Swem \(2017\)](#). Intuitively, cutting-and-running harms an activist's credibility in subsequent interventions. If the market believes that a fund is unable to discipline management, it will be harder for it to convince shareholders in a proxy fight, increasing the cost of actual discipline. Formally, the reputation cost provides an additional degree of market uncertainty that allows us to overcome a discontinuity in the profitability of cut-and-run when blockholder disclosure thresholds bind.

We consider pure strategy Bayesian Nash Equilibrium and obtain the following result:

³³If at $t = 2.3$, liquidity trade was $l_3 \sim U[-yb, yb]$ with $y < 1$ then the information rents from selling stock would be reduced, making cutting-and-running less profitable.

Proposition 11 *In equilibrium, if the disclosure threshold does not bind, the activist does not cut-and-run.*

A formal proof is provided at the end of this section; here we develop the intuition. The activist is time consistent, so our analysis compares the expected profits of each strategy at $t = 2.1$. We denote $E[\Pi_A^I]$ the gross expected profits of intervention at $t = 2.1$ and $E[\Pi_A^R]$ the gross expected profits of cut-and-run at the same period. To establish Proposition 12, we show that $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$. Thus, the activist takes a position and disciplines management if and only if $c \leq E[\Pi_A^I]$, and he never cuts-and-runs.

To ease exposition and analysis, we set $\delta = 1$ so that the bad business plan destroys *all* value, and normalize firm size to one, i.e. $f(k) = 1$. Neither assumption affects the result. Moreover, the reputation cost is irrelevant for Proposition 12 because $0 \geq E[\Pi_A^R]$ implies that it also holds when cutting-and-running has no reputation costs. For simplicity, we temporarily set $\Sigma = 0$. Next, we argue that when prices are consistent with $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$, the activist has no incentives to deviate, so it is an equilibrium. This equilibrium rules out the other equilibrium candidate, $E[\Pi_A^I] \geq E[\Pi_A^R] > 0$, and therefore it is unique.³⁴

Consider $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$ to be an equilibrium candidate. At $t = 2.1$, if the activist takes a position, then $\alpha^* = b$ (recall now that $l_1 \sim U[-b, b]$, not $U[0, b]$). The position is optimal given the tension between positive share value and the cost of information revelation, and therefore it is independent of the subsequent strategy, i.e., (i) act as a mere informed trader, (ii) discipline management, (iii) deviate to cut-and-run. The activist trade is such that with probability 0.5 he conceals and pays $P_{m1} = 1 - z[1 - \lambda G(c_t)]$ for each share. With equal probability the activist is observed and he pays $P_{h1} = 1$ because the market maker believes that either he is acting as a mere informed trader or he will discipline management.

³⁴Deviating to cutting-and-running when prices are consistent with $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$ is more profitable than cutting and running when prices are consistent with $E[\Pi_A^I] \geq E[\Pi_A^R] > 0$. Thus, the two equilibrium candidates are mutually exclusive.

At $t = 2.2$ the activist always reveals his position if it was not involuntarily disclosed at $t = 2.1$. If the activist intends to intervene, then disclosure is inherent in his strategy.³⁵ If, instead, the activist were to deviate and cut-and-run, he must disclose his position to generate the price increase that can make selling the stock prior to intervention potentially profitable.

At $t = 2.3$ the activist may deviate from the equilibrium path and cut-and-run, i.e., to sell stock and not discipline management at $t = 2.4$. The rents from cut-and-run are maximized by selling all stock, i.e., $\mu^* = \alpha^*$, reflecting that l_1 and l_3 have the same distributions. With probability 0.5 the activist conceals and obtains $P_{h3} = 1$ per share. This follows because the market maker believes that the activist kept his position, indicating that either the business plan is good or that he will intervene to discipline management. With equal probability, the market maker observes the activist's sale and then sets a price $P_{l3} = 0$: the market maker now knows with certainty that the business plan is bad and the activist will not discipline management, making the firm worthless.

It follows that the net expected profit per share from deviating to cutting-and-running is $\frac{1}{2} [P_{h3} + P_{l3} - P_{h1} - P_{l1}] < 0$, and therefore it is not a profitable deviation. That $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$ reflects the asymmetry in the activist's cost of revealing information when taking a position and when abandoning the position. While the absence of the activist *does not* imply that the project delivers zero cash flows, cutting-and-running *does*. More specifically, the revenues from secretly selling stock equal the cost of being revealed when acquiring it, i.e., $P_{h3} = P_{h1}$. However, the price paid when acquiring stock secretly outweighs the revenues when cut-and-run is revealed, i.e., $P_{m1} > P_{l3}$ ($P_{l3} = 0$ is only used in this weak way).

The activist's profitability of both intervening $E[\Pi_A^I]$ and cutting-and-running $E[\Pi_A^R]$ is crucially determined by the opportunity to camouflage trade during liq-

³⁵If disclosure was not necessary for intervention, it would still be weakly dominant. In particular, conditional on intervention, the activist is indifferent about disclosing his position. See, e.g., Gantchev (2013), Jiang et al. (2016) or Levit (2017), for work on hedge fund activism that considers activist pressure "behind doors".

uidity shocks. Binding blockholder disclosure thresholds limit the activist position (Corollary 3) and thus increase the likelihood of camouflaging trade. We study the effect of disclosure thresholds when the activist can cut-and-run and obtain the following result:

Proposition 12 *There exists a binding disclosure threshold cutoff $\bar{\alpha}_r < \alpha^*$ such that, in equilibrium,*

- *if a disclosure threshold is weakly higher, i.e., $\bar{\alpha} \geq \bar{\alpha}_r$, the activist does not cut-and-run;*
- *if a disclosure threshold is lower, i.e., $\bar{\alpha} < \bar{\alpha}_r$, the activist cuts-and-runs when the associated reputation cost ς is sufficiently small.*

A binding disclosure threshold reduces the activist's position and thus the probability that his presence is revealed to the market maker. While the threshold yields lower trading profits (Corollary 5), it also reduces information revelation costs and makes cutting-and-running relatively more profitable. In line with the discussion of Proposition 12, a binding threshold increases the probability of both acquiring stock at a low price P_{m1} and selling it at a high price P_{h3} . The cutoff $\bar{\alpha}_r$ satisfies $E[\Pi_A^R] = 0$, with $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$ as a unique equilibrium for $\bar{\alpha} > \bar{\alpha}_r$ and $E[\Pi_A^I] \geq E[\Pi_A^R] > 0$ as a unique equilibrium for $\bar{\alpha} < \bar{\alpha}_r$.

The activist's privately-observed reputation cost ς smoothes out best responses. The activist cuts-and-runs when the associated net profits are both positive, i.e., $E[\Pi_A^R] > \varsigma$, and higher than the net profits of intervening to discipline management, i.e., $E[\Pi_A^R] - \varsigma > E[\Pi_A^I] - c$. Without ς , an inherent discontinuity in activist behavior arises, with resulting consequences for prices, leaving an interval of binding disclosure thresholds for which there is no market equilibrium. This reflects that if the market maker does not believe the activist will cut and run, then absent net order flow below $-b$, the market maker will set a price of 1; but this can make

cutting and running profitable. If the market maker, instead believes that cutting-and-running may occur (then absent ς), the price set when cutting-and-running is concealed will not make it worthwhile.

Beyond the paradox that tight disclosure thresholds can induce the activist to ‘miss-behave’ by cutting-and-running, the results in Proposition 13 do not affect qualitatively the conclusions drawn from the analysis in the main text. The possibility to cut-and-run hurts investors, but benefits the activist for sufficiently tight disclosure thresholds, i.e., $\bar{\alpha} < \bar{\alpha}_r$. If cutting-and-running is profitable, the activist’s incentives to discipline management are reduced, and investors must concede further trading transfers to encourage intervention. Thus, the optimal disclosure threshold policy for investors is higher, and investment is reduced. In contrast the activist can gain more from tighter disclosure thresholds.

Proof of Proposition 12

Assume that there is no reputation cost for cut-and-run—if the result holds when $\Sigma = 0$, it must hold for any $\Sigma > 0$. We consider $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$ an equilibrium candidate, and show that the activist has no incentives to deviate. This rules the other equilibrium candidate. In particular, if $E[\Pi_A^I] \geq 0 \geq E[\Pi_A^R]$ is an equilibrium, then $E[\Pi_A^I] > E[\Pi_A^R] > 0$ cannot be an equilibrium because the corresponding market prices make the activist strictly worse off. Moreover $E[\Pi_A^R] \geq E[\Pi_A^I]$ can never be an equilibrium because expected prices from cutting-and-running at $t = 2.3$ must be strictly lower than the secure cash flows from discipline, which have been normalized to one. We solve recursively.

Intervention ($t = 2.4$). Liquidity shocks at $t = 2.1$ and $t = 2.3$ have the same distribution so we conjecture and later verify that if the activist cuts-and-runs he sells all stock, i.e., $\mu \in \{\alpha^*, 0\}$. If the activist has α^* shares and the business plan is bad ($m = 0$), he intervenes to discipline management if and only if $\alpha^* - c \geq 0$.

Exit ($t = 2.3$). If the activist has a position and the business plan is good, he does not sell stock—he acts as a standard informed trader. If the business plan is bad, he cuts-and-runs if and only if $c > c_{t(2.3)}$, where $c_{t(2.3)}$ is derived below. Given the equilibrium candidate $E[\Pi_A^I] > 0 > E[\Pi_A^R]$, the market maker conjectures $\hat{\mu} = 0$ and his pricing rule is

$$\begin{aligned} P(\omega) &= P_{l3} \equiv 0 & \text{if } \omega < -b \\ P(\omega) &= P_{h3} \equiv 1 & \text{if } \omega > -b \end{aligned} \quad (1.65)$$

Here, P_{h3} captures the equilibrium path given the activists presence, which is either due to the good business plan, or because the plan is bad and he plans to discipline management. P_{l3} accounts for a potential deviation where the activist cuts-and-runs. A net order flow $\omega < -b$ would reveal activist sales and indicate with certainty that the project yields zero cash flows—the plan is bad and that the activist will not intervene.

If the activist deviates to cut-and-run, then with probability $\int_{-b+\mu}^b \frac{1}{2b} dl = \frac{2b-\mu}{2b}$ he camouflages his sales μ and obtains P_{h3} for each share. Thus, cut-and-run has expected payoff $\left(\frac{2b-\mu}{2b}\right) \mu$, which is maximized for $\mu = b$. Therefore, $\mu^* = \min\{b, \alpha^*\}$. We conjecture and verify that $\alpha^* = b$. If the conjecture is correct, then cut-and-run is such that $\mu^* = b$, the activist conceals stock sales with probability 0.5, and obtaining expected revenues $b/2$.

It follows that when the business plan is bad, the activist decides whether intervene in the next period and obtain $\alpha - c$ or deviate cut-and-run, which has expected payoff $b/2$. The activist deviates to cut-and-run if and only if $c > c_{t(2.3)}$, where $c_{t(2.3)} = \alpha - \frac{b}{2} = \frac{b}{2}$.

Disclosure ($t = 2.2$). The activist always discloses his position. It is necessary in the case of intervention. If, instead, he plans to cut-and-run, disclosure yields the increase in price that can make this strategy profitable.

Entry ($t = 2.1$). The market maker's conjecture about activist position is $\hat{\alpha}$. The

net order flow ω either (i) equals liquidity trade because the activist does not acquire shares, i.e., $\omega = l$; or (ii) it is the combination of liquidity trade and the activist position, i.e. $\omega = l + \hat{\alpha}$. The activist acquires shares when either he observes the good business plan, or he observes the bad business plan and intervention profitable, i.e. $c \leq c_t$ where c_t is derived below. The expected value of the project is

$$E_{2.1}[V] = \frac{x(\omega)(1-z)(1-\lambda) + x(\omega - \hat{\alpha})(1-z)\lambda + x(\omega - \hat{\alpha})z\lambda G(c_t)}{\left[\begin{array}{c} x(\omega)(1-z)(1-\lambda) + x(\omega - \hat{\alpha})(1-z)\lambda + x(\omega - \hat{\alpha})z\lambda G(c_t) \\ + x(\omega)z(1-\lambda) + x(\omega)z\lambda(1-G(c_t)) \end{array} \right]}$$

If the market maker observes $\omega < -b + \hat{\alpha}$, then $x(\omega - \hat{\alpha}) = 0$ and $x(\omega) = 1/2b$. If $\omega > b$, then $x(\omega - \hat{\alpha}) = 1/2b$ and $x(\omega) = 0$. If $\omega \in [-b + \hat{\alpha}, b]$, then $x(\omega - \hat{\alpha}) = x(\omega) = 1/2b$. The market maker's pricing rule satisfies

$$\begin{aligned} P(\omega) &= P_{l1} \equiv \frac{(1-z)(1-\lambda)}{(1-\lambda)+z\lambda(1-G(c_t))} & \text{if } \omega < -b + \hat{\alpha} \\ P(\omega) &= P_{m1} \equiv 1 - z(1 - \lambda G(c_t)) & \text{if } \omega \in [-b + \hat{\alpha}, b] \\ P(\omega) &= P_{h1} \equiv 1 & \text{if } \omega > b \end{aligned} \quad (1.66)$$

With probability $\int_{-b}^{b-\alpha} \frac{1}{2b} dl = \frac{2b-\alpha}{2b}$ the activist camouflages his position α . The gross expected profits from intervention read $\left(\frac{2b-\alpha}{2b}\right) \alpha[1 - P_{m1}]$ and are maximized for $\alpha^* = b$. This verifies our two previous conjectures: $\alpha^* = \mu^* = b$. It follows that with probability 0.5 the activist camouflages his stock purchase, and the expected gross payoff from taking a position and intervene if the business plan is bad reads $E[\Pi_A^I] = z[1 - \lambda G(c_t)] \frac{b}{2}$.

The activist takes a position after observing the bad business plan if and only if $c \leq c_t$, where $c_t = z[1 - \lambda G(c_t)] \frac{b}{2}$. The relation $c_t < c_{t(2.3)}$ confirms that the activist is time consistent. Put differently, if the activist takes a position at $t = 2.1$ with the intention to discipline, he does not cut-and-run at $t = 2.3$.

Last, the discussion of Proposition 12 shows that cut-and-run is unprofitable at $t = 2.1$.

Proof of Proposition 13

We introduce binding disclosure thresholds $\bar{\alpha} < \alpha^* = b$ and show that there exists $\bar{\alpha}_r$ such that (i) cut-and-run is never profitable for $\bar{\alpha} \geq \bar{\alpha}_r$; (ii) cut-and-run can be profitable for $\bar{\alpha} < \bar{\alpha}_r$. The analysis for periods $t = 2.2$ and $t = 2.4$ mirrors that in the proof of Proposition 12, so we focus on equilibrium outcomes at $t = 2.1$ and $t = 2.3$, i.e., the trading periods.

i) There exists $\bar{\alpha}_r$ such that cut-and-run is not profitable for $\bar{\alpha} \geq \bar{\alpha}_r$. We show that $E[\Pi_A^R] \leq 0 \iff \bar{\alpha} \geq \bar{\alpha}_r$, so suppose there is no reputation cost, i.e. $\Sigma = 0$. In line with the proof of Proposition 12, we consider $E[\Pi_A^I] > 0 \geq E[\Pi_A^R]$ as an equilibrium candidate and show that there are no incentives to deviate. From that proof, we also know that a disclosure threshold $\bar{\alpha}$ binds if and only if $\bar{\alpha} < b$, and that the most profitable deviation involves selling all stock at $t = 2.3$. Thus, deviating when a disclosure threshold binds implies $\mu = \alpha = \bar{\alpha} < b$.

Exit ($t = 2.3$). The market maker sets prices in (1.65). If the activist deviates to cut-and-run he sells $\bar{\alpha}$ shares and camouflages with probability $\int_{-b+\bar{\alpha}}^b \frac{1}{2b} dl = \frac{2b-\bar{\alpha}}{2b}$. Thus, the expected payoff of deviating to cut-and-run is $(\frac{2b-\bar{\alpha}}{2b}) \bar{\alpha}$. The payoff of intervention at $t = 2.3$ is $\bar{\alpha} - c$, so the activist cuts-and-runs if and only if $c > c_{t(2.3)}$ where $c_{t(2.3)} = \frac{\bar{\alpha}^2}{2b}$.

Entry ($t = 2.1$). The market maker sets prices in (1.66). With probability $\int_{-b}^{b-\bar{\alpha}} \frac{1}{2b} dl = \frac{2b-\bar{\alpha}}{2b}$ the activist camouflages his position $\bar{\alpha}$. The expected gross profits from intervention are $E[\Pi_A^I] = \bar{\alpha} (\frac{2b-\bar{\alpha}}{2b}) [1 - P_{m1}]$. The characterization of P_{m1} in (1.66) and the relation $E[\Pi_A^I] = c_t$ pin down the intervention cost cutoff $c_t = z [1 - \lambda G(c_t)] (\frac{2b-\bar{\alpha}}{2b}) \bar{\alpha}$.

The expected gross profits from cut-and-run are

$$E[\Pi_A^R] = \bar{\alpha} \left(\frac{2b-\bar{\alpha}}{2b} \right) P_{h3} - \bar{\alpha} \left(\frac{2b-\bar{\alpha}}{2b} \right) P_{m1} - \bar{\alpha} \left(\frac{\bar{\alpha}}{2b} \right) P_{h1}, \quad (1.67)$$

where prices are given by both (1.65) and (1.66). Using $P_{h3} = P_{h1}$, algebra confirms

that $E[\Pi_A^R] \leq 0$ if and only if $\bar{\alpha} \geq \bar{\alpha}_{r(i)}$, where

$$\bar{\alpha}_{r(i)} = \frac{2b(P_{h3} - P_{m1})}{2P_{h3} - P_{m1}}. \quad (1.68)$$

Note that $c_{t(2.3)} \geq c_t$ if $\bar{\alpha} \geq \bar{\alpha}_{r(i)}$, which verifies time consistency.

ii) Cut-and-run might be profitable for $\bar{\alpha} < \bar{\alpha}_r$. We now show that $E[\Pi_A^I] > E[\Pi_A^R] > 0$ is the unique equilibrium for disclosure thresholds such that $\bar{\alpha} < \bar{\alpha}_{r(ii)}$, where $\bar{\alpha}_{r(ii)} = \bar{\alpha}_{r(i)}$ when $E[\Pi_A^R] = 0$ and $\bar{\alpha}_{r(ii)} < \bar{\alpha}_{r(i)}$ for $E[\Pi_A^R] > 0$. The reputation cost is now relevant for the activist's strategy, so we consider $\Sigma > 0$.

To ease exposition, we define the following probabilities:

$$\begin{aligned} \Pr^I &\equiv \Pr \left[E[\Pi_A^I] - c \geq E[\Pi_A^R] - \varsigma \mid E[\Pi_A^I] - c \geq 0 \right] \times \Pr \left[E[\Pi_A^I] - c \geq 0 \right] \\ \Pr^R &\equiv \Pr \left[E[\Pi_A^R] - \varsigma > E[\Pi_A^I] - c \mid E[\Pi_A^R] - \varsigma > 0 \right] \times \Pr \left[E[\Pi_A^R] - \varsigma > 0 \right] \end{aligned}$$

Here, P^I is the probability that the activist intervenes to discipline management after observing the bad business plan ($m = 0$), i.e., when intervention is both profitable and more profitable than cut-and-run. Analogously, after observing the bad business plan, cut-and-run is optimal with probability P^R . The probability that the activist does not take a position after observing the bad business plan is $1 - P^I - P^R$.

Exit ($t = 2.3$). Suppose the activist has a position in the company. Let $\hat{\mu}$ be the market maker's conjecture about the activist's sales if he cuts-and-runs. The net order flow ω either equals liquidity trade because the activist does not cut-and-run, i.e., $\omega = l_3$; or it is the sum of liquidity trade and activist sales, i.e. $\omega = l_3 - \mu$. The expected value of the project is

$$E_{2.3}[V] = \frac{x(\omega)(1 - z) + x(\omega)z \Pr^I}{x(\omega)(1 - z) + x(\omega)z \Pr^I + x(\omega + \hat{\mu})z \Pr^R}.$$

If the market maker observes $\omega < -b$, then $x(\omega + \hat{\mu}) = 1/2b$ and $x(\omega) = 0$. If $\omega > b - \hat{\mu}$, then $x(\omega + \hat{\mu}) = 0$ and $x(\omega) = 1/2b$. If $\omega \in [-b, b - \hat{\mu}]$, then $x(\omega + \hat{\mu}) = x(\omega) = 1/2b$. Accordingly, pricing is given by

$$\begin{aligned} P(\omega) &= P_{l3} \equiv 0 && \text{if } \omega < -b \\ P(\omega) &= P_{m3} \equiv \frac{(1-z)+z\Pr^I}{(1-z)+z[\Pr^I+\Pr^R]} && \text{if } \omega \in [-b, b - \hat{\mu}] \text{ .} \\ P(\omega) &= P_{h3} \equiv 1 && \text{if } \omega > b - \hat{\mu} \end{aligned} \quad (1.69)$$

From the proof of Proposition 12, the profits from cut-and-run are maximized by selling all shares—and hence, given the binding disclosure threshold, $\mu = \alpha = \bar{\alpha}$. The activist camouflages his sales with probability $\int_{-b+\bar{\alpha}}^b \frac{1}{2b} dl = \frac{2b-\bar{\alpha}}{2b}$, so the expected gross payoff from cut-and-run is $\left(\frac{2b-\bar{\alpha}}{2b}\right) \bar{\alpha} P_{m3}$ where P_{m3} is given by (1.69).

Entry ($t = 2.1$). The market maker's conjecture about the activist's trade is $\hat{\alpha}$. The net order flow ω either equals liquidity trade $l_1 = \omega$; or it is the sum of liquidity trade and the activist's purchases, $\omega = l_1 + \hat{\alpha}$. The activist acquires shares when either he observes the good business plan, or he observes the bad business plan and wants to intervene or cut-and-run. Thus,

$$E_{2.1}[V] = \frac{x(\omega)(1-z)(1-\lambda) + x(\omega - \hat{\alpha})(1-z)\lambda + x(\omega - \hat{\alpha})z\lambda\Pr^I}{\begin{bmatrix} x(\omega)(1-z)(1-\lambda) + x(\omega - \hat{\alpha})(1-z)\lambda + x(\omega - \hat{\alpha})z\lambda\Pr^I \\ + x(\omega - \hat{\alpha})z\lambda\Pr^R + x(\omega)z\lambda[1 - \Pr^I - \Pr^R] + x(\omega)z(1-\lambda) \end{bmatrix}}.$$

If the market maker observes $\omega < -b + \hat{\alpha}$, then $x(\omega - \hat{\alpha}) = 0$ and $x(\omega) = 1/2b$. If $\omega > b$, then $x(\omega - \hat{\alpha}) = 1/2b$ and $x(\omega) = 0$. If $\omega \in [-b + \hat{\alpha}, b]$, then $x(\omega - \hat{\alpha}) = x(\omega) = 1/2b$. Thus

$$\begin{aligned} P(\omega) &= P_{l1} \equiv \frac{(1-z)(1-\lambda)}{(1-z)(1-\lambda)+z(1-\lambda)+z\lambda[1-\Pr^I-\Pr^R]} && \text{if } \omega < -b + \hat{\alpha} \\ P(\omega) &= P_{m1} \equiv (1-z) + z\lambda\Pr^I && \text{if } \omega \in [-b + \hat{\alpha}, b] \text{ ,} \\ P(\omega) &= P_{h1} \equiv \frac{(1-z)+z\Pr^I}{(1-z)+z[\Pr^I+\Pr^R]} && \text{if } \omega > b \end{aligned} \quad (1.70)$$

where $P_{m1} < P_{h1}$.

With probability $\int_{-b}^{b-\bar{\alpha}} \frac{1}{2b} dl = \frac{2b-\bar{\alpha}}{2b}$ the market maker camouflages his position and pays P_{m1} for each share. Thus, the gross payoff from cut-and-run is

$$E[\Pi_A^R] = \bar{\alpha} \left(\frac{2b-\bar{\alpha}}{2b} \right) P_{m3} - \bar{\alpha} \left(\frac{2b-\bar{\alpha}}{2b} \right) P_{m1} - \bar{\alpha} \left(\frac{\bar{\alpha}}{2b} \right) P_{h1}. \quad (1.71)$$

Algebra manipulation when $P_{m3} = P_{h1}$ reveals that $E[\Pi_A^R] > 0$ if and only if $\bar{\alpha} < \bar{\alpha}_{r(ii)}$, where

$$\bar{\alpha}_{r(ii)} = \frac{2b(P_{m3} - P_{m1})}{2P_{m3} - P_{m1}},$$

where the prices are given in (1.69) and (1.70).

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Chapter 2

Takeover resistance: a global games analysis

2.1 Introduction

In 2009 Kraft Foods, a US company, launched a hostile bid for Cadbury, the UK-listed chocolatier. Cadbury’s managerial board declared the offer “unattractive” even though it offered a premium on market prices. The British government and some unions publicly stood against the takeover, with the business secretary warning Kraft that it could meet “huge opposition” from the British Government. Despite the political distaste for the deal, Kraft eventually won the Cadbury’s board’s blessing. The agreement was acknowledged to be unpopular amongst many Cadbury’s shareholders, but nonetheless the board’s approval led to the takeover succeeding. Sir Roger Carr, Cadbury’s Chairman, admitted that the high shareholder turnover during the takeover negotiations led him to close the deal:¹

“In the final analysis of the deal, it was the *shift in the shareholder*

¹Press coverage includes: Wiggins, J. (2010). The inside story of the Cadbury takeover. *Financial Times*, [online]. Available at: <http://www.ft.com/cms/s/2/1e5450d2-2be5-11df-8033-00144feabdc0.html> [Accessed 17 Jan. 2016]; or Moeller, S. (2012). Case Study: Kraft’s takeover of Cadbury. *Financial Times*, [online]. Available at: <http://www.ft.com/cms/s/0/1cb06d30-332f-11e1-a51e-00144feabdc0.html#axzz3xVfEY4Dk> [Accessed 17 Jan. 2016]

register that lost the battle for Cadbury.” Roger Carr, former Chairman of Cadbury (emphasis added).²

Managerial resistance is acknowledged to play an important role in the market for corporate control. While takeover resistance might be motivated by managerial entrenchment, the fiduciary duty to act on behalf the shareholders’ interests gives credibility to Boards that recommend against a takeover. The ability of a Board to resist an offer by arguing that it undervalues the company is tempered however by share sales during the offer period for at least three reasons. Firstly shareholder sales can be regarded as a vote with their feet against corporate management (Edmans 2009; Admati and Pfleiderer 2009). Board’s recommendations opposing takeover bids are often challenged in courts, and it would be hard for Directors to argue the offer was derisory when shareholders are selling not only for less than some future promised price, but typically for less than the takeover price offered. Secondly a substantial amount of this stock is acquired by institutional investors such as risk-arbitrageurs (Cornelli and Li 2002) and hedge fund activists (Jiang et al. 2016; Corum and Levit 2017) that exert additional pressure on the Board for the takeover to go through. Finally, not all shareholders are equally sophisticated, and so prominent or institutional shareholders that sell are sending a negative signal to other owners which will make these shareholders in turn less likely to support a Board’s rejection of the offer. Given that it is damaging to directors’ careers to suffer a rebellion of shareholders, this once again makes it less likely the takeover will be rejected.

All of these channels act to lower the willingness of a Board to reject a takeover as a function of the volume of existing shareholders who sell up; and this effect remains unexplored. Shareholders therefore have a strategic role to play in the success or otherwise of takeovers via the pressure they create on the Board.

²‘UK takeover threshold should be raised, says ex-Cadbury chairman Roger Carr,’ Daily Telegraph, 10 February 2010.

The extent of takeover resistance is often reinforced by external pressure exerted by the political system, labour, the media and other stakeholders whose interests are aligned with those of the incumbent managers (Hellwig, 2000). Governments in particular use coercion of Boards as a tool of industrial policy when formal approaches, such as nationalisation or the purchasing of a controlling stake, are deemed too expensive or unjustifiable.³

This paper formally analyses the takeover process from this novel perspective. Boards' resistance to takeovers is decreasing in the premium offered by the bidder and the ability of a Board to reject a takeover is reduced in the proportion of shareholders who sell in the offer period: after the bid and before the formal decision of the Board. However Government pressure may bolster the Board's resistance. Sophisticated shareholders therefore face an environment of strategic substitutes: if enough other shareholders sell the Board will find themselves unable or unwilling to reject the takeover and so a shareholder prefers to hold her shares so as to gather the full takeover premium. In contrast, if many other shareholders do not sell early then the takeover will likely fail. In this case a shareholder would rather sell in the offer period so as to profit from the raised market price before the stock price returns to its original value after the rejection of the takeover. In either case sophisticated shareholders wish to coordinate against each other and selling is akin to a *public good* to which no one wishes to contribute, but everyone hopes others will.

In equilibrium both the stock price and stock sales at the interim are determined endogenously. Thus we have a shareholder-led coordination feedback loop between stock sales, the interim stock price and the final corporate outcome. This feedback loop is distinct from the current scholarly focus on the learning feedback loop.⁴ The simplest model of this situation would generate multiple equilibria ra-

³Dinc and Erel (2013) include "moral persuasion" and the arbitrary use of financial regulation as government instruments to deter takeovers of domestic companies from foreign bidders in the context of economic nationalism.

⁴See Bond et al. (2012) for a survey.

tionalising both holding and selling and so prevent analysis.⁵

We enrich the framework by using a global games approach. We suppose that the level of managerial resistance is private information about which different market participants receive a noisy signal. As a result both market makers and shareholders face an inference problem embedded within the game of strategic substitutes described above. Solving global games with strategic substitutes is more challenging than the normal strategic complements framework. Nonetheless we can demonstrate that a unique threshold equilibrium exists allowing us to study the takeover process.

The fundamentals of our analysis will be the bid premium, the expected Board level of resistance, and the influence of shareholder sales on the Board's verdict. We study how changes in these parameters affect the main outcomes of our analysis: the volume of shareholders who choose not to wait for the Board's recommendation but rather sell early; the market price these shareholders receive during the offer period; and ultimately the probability a takeover succeeds.

Our study provides a full comparative statics analysis of the interplay of these variables. We would draw attention to two key results.

The first result is that we find that limiting the voting rights of new shareholders, or reducing the directors' vulnerability to legal challenge arising from their takeover rejection decision, reduces the probability of a successful takeover but it *increases* the number of existing shareholders who choose to sell. The direct effect of these changes is for the probability of a successful deal to decline. This implies that the market price of the stock in the offer period will fall. This would seem to create a tension: the lower probability of a deal encourages shareholders to sell and profit early; however the lower interim share price deters them from doing so and

⁵If all other shareholders will hold their stock then the takeover will fail for sure, but in this case the price of the stock during the offer period is unchanged from pre-bid levels. Hence holding the stock is an equilibrium. If instead all other shareholders sell early then the takeover will definitely succeed, which implies the price during the offer period will rise to match the bid price, and so shareholders gain nothing by waiting: hence selling the stock early is also an equilibrium.

encourages shareholders instead to hold out for the chance of the full offer premium. In our analysis this tension resolves itself in favour of selling early and the proportion of shareholders selling during the offer period increases. The reason for this is subtle: it follows because some shareholders in our model are sophisticated and so have slightly more precise information than other market participants. This is natural if the shareholders have greater experience of management and so are better able to judge their resistance to the takeover. As a result shareholders' strategies are more sensitive to new information than the market price, and this implies that the fall in the interim price is relatively small and so results in early sale.

This result has an immediate corollary. A lively debate in corporate law discusses the merits of respecting Boards' discretion to recommend for or against takeover offers; that is courts respecting the "*business judgment rule*" in takeover situations.⁶ In practice this rule insulates the directors from legal challenge and so allows the Board to ignore the fact that exiting shareholders have demonstrated that they do not agree that the firm has an even more valuable future than that offered by the acquirer. As noted above, pressure to agree to a takeover is also exerted by the new purchasers of shares. Commentators have expressed concern that such new shareholders do not have the long-term interests of the company at heart, and so weakening the voting power of new shareholders is also being actively considered.⁷ Our analysis highlights a perhaps unexpected consequence of such interventions: weakening the influence of new shareholders or affording the Board greater powers of court-protected discretion encourages more existing shareholders to sell early, thereby increasing the turnaround in the register of the target company. Hence absent strong respect for new shareholders, or under a wide interpretation of the business judgment rule, more, not fewer, shareholders will seek to take profits early

⁶See [Bebchuk et al. \(2002\)](#) and the academic debate that follows.

⁷The Aspen Institute in the US propose that shareholders should be able to vote only after a minimum holding period. The European Commission are considering increasing the voting weight of shareholders who are long-term holders of the stock. (See *Brussels aims to reward investor loyalty*, Financial Times, Jan 23, 2013). In the UK the Takeover Panel also considered disenfranchising new shareholders following the Cadbury takeover.

and sell out of a stock as soon as a takeover offer is received.

Our second key result studies when political pressure on a Board is likely to be most effective, and so when it is most likely to occur. We equate the marginal benefit to a government of encouraging takeover resistance with the extent to which the probability of a takeover succeeding is reduced. Our result offers a salutary warning. Jurisdictions which have the greatest respect for shareholders are the ones in which the marginal benefit to increasing resistance is the greatest. In countries which pride themselves on shareholder power, perhaps like the UK and the US, the Board's decision is most sensitive to the sale decisions of shareholders during the offer period. Government intervention to raise the expected resistance of the Board will act to lower the benefit of selling early. We show that the interim price change will not compensate fully so that the number of shareholders who sell in the offer period can be reduced. This in turn weakens the pressure on the Board to agree to the takeover and allows the government to secure a more likely rejection of the bid.

The paper is structured as follows. The literature review follows. The model is formally presented in Section 2.2, and is solved in Section 2.3 which includes analysis of the information structure and uniqueness conditions. Section 2.4 then studies the solution exploring comparative statics, explores the marginal benefit to outside parties of pressurising the Board, and extends the analysis to endogenise the bid premium. Section 2.5 considers the empirical predictions of our analysis and studies the robustness of our model. Section 2.6 concludes with all proofs contained in the Technical Appendix (starting on page 119).

2.1.1 Related Literature

Our analysis contributes to the study of coordination problems among target shareholders in takeovers. The seminal analysis of Grossman and Hart (1980) began a rich research endeavour studying the returns to corporate takeovers when the target firms' shareholders are strategic. The core economic issue studied is that dispersed

shareholders have an incentive to demand rents before agreeing to sell their shares to a corporate raider, so discouraging profitable takeovers. In the intervening thirty years our understanding of when raiders can make money has developed considerably (Tirole, 2010, Marquez and Yilmaz, 2008, 2012; Ekmekci and Kos, 2016). Our work makes distinct contributions to this literature along two dimensions. Firstly this literature has explored the decision of existing stock holders to either accept or reject a bidder’s offer. The stock market during the takeover is not modelled and so the feedback effect between the stock sales during the offer period and the final corporate outcome is not studied. Secondly these models focus on the shareholders and so decide not to model the role of the Board of the target firm; thus managerial resistance has not been theoretically explored in this context.

Investors that acquire stock during the offer period play a critical role in the coordination problem characterized here. We parsimoniously assume that the Board’s ability to resist the takeover is tempered by the volume of stock acquired by new shareholders. Cornelli and Li (2002) show that risk-arbitrageurs make excess returns by acquiring stock during the offer period because they increase the probability of the takeover. Corum and Levit (2017) characterize the complementarity of bidders and hedge fund activists that exert pressure for the takeover to go through. Our analysis complements the study of these mechanisms by recognizing that they create a coordination problem among existing shareholders, and that stock sales are necessary for new shareholders to influence takeover outcomes.

Our paper is related to Levit (2017), which studies the ability of a Board to educate and advise shareholders as to the desirability of a takeover offer. Levit models the communication between shareholders and the Board, and characterizes the interplay between the takeover premium offered by the bidder and the Board’s ability to influence shareholders. We simplify these elements in a setting where the Board’s advice is deterministic for the takeover outcome. However we alter the scope of the analysis in two dimensions. The first is that we drop the assumption

that a bidder's offer is available to any shareholder unilaterally. Takeover offers are typically conditional on the proportion of shareholders agreeing so as to ensure that the acquirer can secure sufficient control for their purposes; in the case of private equity purchases full control is required to allow the bidder to avoid minority shareholding restrictions. We will discuss these arrangements in the model below and explain why they give the Board's recommendation significant prominence. Secondly we introduce the stock market into the analysis thus endogenising the sale price which a shareholder must accept before a deal is consummated. Thus, the two papers offer a novel perspective to the study of takeovers and characterize different mechanisms that influence final outcomes.

Our work is part of the current research effort which studies how stock markets can alter business decisions and so generate a feedback loop back to the stock market. We have an entirely innovative channel for this feedback loop. The key channel driving the existing research might be characterised as a speculator-led learning feedback loop ([Edmans et al., 2015](#); [Dow et al., 2016](#); [Foucault and Frersard, 2014](#)): managers seek to infer speculators' information from the stock market variables of price and order flow. Our feedback channel is instead a shareholder-led coordination feedback loop: firm shareholders seek to coordinate on enough of them selling out of a stock that management are pressurised into following a given course of action. This channel is fundamentally different in the tools required to study it and in its characteristics. On techniques, as we study a coordination issue we develop global games techniques to analyse the multiplayer interaction between shareholders; whereas inference techniques are required in the work on the speculator-led learning feedback loop. Secondly, to see the different characteristics of the two mechanisms suppose management consider an action which shareholders believe is a bad idea – such as potentially rejecting a good takeover offer. Under the speculator-led learning feedback loop speculators will refrain from buying the stock and this will act to deliver a low share price; observing the low share price the management will infer

the bad news and reconsider the action. Under the shareholder-led coordination feedback loop a sufficient volume of existing shareholders walking away from the firm will cause management to reconsider the action; the share price plays no direct role, though it will of course change as shareholders sell.

We adopt the information structure of a global game to study the shareholder coordination issues (Bebchuk and Goldstein, 2011; Morris and Shin, 2003). However our setting differs from most of the previous analyses in that agents' actions are strategic substitutes. The case of strategic substitutes has not been as thoroughly studied as games with strategic complementarities. Hence, while the properties of equilibria are analysed in Morris and Shin (2005) and Harrison (2003) among others, we find little application of this case. Nonetheless by using the monotonicity of the signal distribution functions we can establish the existence and uniqueness of a threshold equilibrium.

The information structure of the global game is used to motivate trade. In this aspect our paper is similar to Brav et al. (2016), where the information asymmetries about the resistance of a managerial Board provide a rationale for trading. In their setting heterogeneous outside investors receive private signals that give them an information advantage. We differ from this approach and follow the standard assumption in the literature of takeovers that shareholders, this is the supply side of the market, are better informed about the stock value (see Ekmekci and Kos, 2016 and references therein). In Appendix 2.8.2 we reverse the information structure to test the implications of this assumption and show that most of our results are robust. We differ from the traditional Kyle (1985) trading environment in that our market makers do not observe the order flow. While it is possible to relax this assumption by introducing a proportion of noise traders, our assumption does not affect the results qualitatively and provides a more tractable setting.

2.2 The Model

We consider a target company which has a market value V . The target company is owned by a continuum of shareholders and represented by a Board. At the beginning of the game the target receives an offer $P > V$ from the bidder. The actors we model in the takeover process are therefore the Bidder, the Target Board, the sophisticated shareholders, and the market makers. We introduce each of these in turn.

2.2.1 The Bidder

If the bidder can acquire sufficient control of the target then she can generate value of $W \geq P$ from the target's assets. In our benchmark analysis we study the target's behaviour taking the offer price P as given. This is accurate when the acquirer loses the ability to unilaterally set P , such as in a competitive bidding or auction context. We extend the model to endogenise the bid price in Section 2.4.3.

If however the bidder were not to secure sufficient of the target then her valuation for the target is assumed to be weakly less than V and so she would not wish to buy the company in this case. Private equity buyers typically require complete control to allow them to take a target private. In the UK a contractual offer, equivalent to a tender offer in the US, only becomes binding for all stockholders when accepted by 90% of the shareholder equity. If the Board recommend the offer then an alternative legal structure can be used (known as a scheme of arrangement) in which 75% of stock must accept the offer for it to be binding on all shareholders.⁸ If the bidder secures less than complete control then she will have to respect minority shareholder rights. This lowers the freedom the acquirer has to act and so lowers the value of the firm to the bidder. In reality therefore bids are typically conditional on the proportion of shares accepting. We reflect this here by assuming that the bidder's offer will only stand if she secures acceptance from a large enough proportion

⁸For more on the rules of takeovers see, for example, "Public Company Takeovers in the United Kingdom: A Guide for US Private Equity Acquirers", by Squire & Sanders, or other similar legal publications.

of the target shareholders. We will make this precise when we introduce the target's shareholders. First however we present the target Board.

2.2.2 The Board

The Board of the target receive the offer at $t = 0$, and must decide how to respond to the offer at $t = 2$. We model the Board as recommending acceptance of the offer if:

$$P - V \geq \theta - \kappa \cdot \rho \quad (2.1)$$

The left hand side of (2.1), $P - V$, is the bid premium. The Board's decision rule is therefore more complicated than recommending acceptance only if the bid premium is positive. Our analysis captures, in a parsimonious way, two salient features of the takeover process.

The parameter θ is a measure of the Board's resistance to the takeover. The higher is this parameter, the more reluctant the Board is to recommend the offer to shareholders. θ may capture the value to the Board of private rents from control; or it may capture the extent of political pressure to which the Board is exposed. We assume that θ is private information to the Board. Nature draws θ according to $\theta \sim N(y, 1/\tau_\theta)$ where y is the publicly known expected value, and the precision with which θ is known is τ_θ . We will subsequently analyse a Board subject to political pressure by studying the incentives to a government to move the expected level of resistance: y .

The term $\kappa \cdot \rho$ captures the effect of shareholders selling out of the firm during the offer process on the Board. ρ is the proportion of shareholders who sell in the interim period between the offer being received and the Board announcing its recommendation; it is determined endogenously. Equivalently, ρ represents the shareholder turnover at the interim. The interim period is often referred to as the offer period. $\kappa > 0$ is a scaling factor. The larger the proportion of the shareholders

who sell during the offer period the harder it is for a Board to subsequently reject the offer. This explicit role for shareholder action is a hitherto unstudied part of the takeover process; it is however an important part for at least three reasons:

1. Firstly, Boards are under a fiduciary duty to represent shareholders' best interests.⁹ It is a Board's duty to reject a takeover offer which undervalues the long-run value of the company. However, the greater the proportion of shareholders selling out for less than the acquirer has offered, the more vulnerable individual directors will feel should there be a legal challenge to their recommendation. Shareholder sales can be interpreted as a vote with their feet against the Board's standing (Edmans 2009; Admati and Pfleiderer 2009). It would be hard to argue that the offer undervalues the long-run value of the company when a significant number of shareholders are willing to sell at an interim price which will typically be, not only below some aspirational long-run value, but even below the offered acquisition price.
2. Secondly, as the Chairman of Cadbury publicly argued, in any shareholder vote on the offer buyers of the shares during the offer period, and so buyers of the stock at a premium to the pre-merger share price, could be expected to vote for the acquisition. One might counter that the existing owners of the stock would also vote for the acquisition. However the buyers of the shares are unlikely to have the same influence as the sellers. Often times the buyers of shares during an offer period are risk-arbitrageurs and hedge fund activists who have been documented as exerting pressure on the Board and increasing the probability of a takeover deal being consummated (Larcker and Lys 1987; Cornelli and Li 2002; Jiang et al. 2016; Corum and Levit 2017). Alternatively, as we already noted, policy attention has focused on whether new owners should have their

⁹In the US, the fiduciary duty developed from the 1919 Dodge v. Ford Motor Co. decision, which determined that a business corporation exists primarily for the profit of the stockholders. This interpretation has been reconfirmed multiple times since, including, for example, in eBay Domestic Holdings Inc. v. Newmark.

voting power reduced for a period, to make them less influential. These two effects are captured by the scaling parameter κ .

3. Finally, shareholders are not all likely to be equally sophisticated. If sophisticated existing shareholders vote with their feet by selling, so some of the remaining shareholders will interpret the decision to sell as a loss of confidence in the future of the firm as an independent entity. This will make such shareholders less likely to follow a Board's recommendation to reject the takeover offer. This will make Boards less likely to reject the takeover offer in the first place.

All of these effects imply that as existing shareholders sell, the weaker becomes the basis a Board has for rejecting a takeover offer, and so acceptance becomes more likely. Our decision rule (2.1) is designed to parsimoniously capture these effects. The Cadbury-Kraft example of the introduction demonstrated these were key: *“the shift in the shareholder register ... lost the battle for Cadbury.”*

The presence of the scaling factor κ captures the strength of the pressure that shareholders' sales exert on the Board. The greater is κ the more likely a Board is to recommend deal acceptance for any given proportion of shareholders selling during the offer period. Our analysis will therefore allow us to study the impact of shareholder strength on stock sales at the offer period and on equilibrium takeover premia.

2.2.3 Shareholders

We assume that a proportion δ of shareholders are *sophisticated*, these will receive information and act strategically during the takeover game, the remaining shareholders are *naïve*. During the offer period ($t = 1$) each sophisticated shareholder i receives a private signal x_i as to the level of the Board's resistance. This signal is specific to each shareholder and is independently drawn from the distribution $x_i \sim N(\theta, 1/\tau_\epsilon)$.

Hence τ_ϵ is the precision of each shareholder’s signal. Upon updating beliefs, each sophisticated shareholder either *sells* his stock or *holds* it until the Board makes its recommendation. Naïve shareholders do not receive private signals as to the Board’s type, nor do they consider selling their stock during the offer period. The shareholder turnover ρ weakening the Board’s resistance —see (2.1), is captured by

$$\rho = \delta\gamma. \tag{2.2}$$

and is also referred to as stock sales. Here, $\gamma \in [0, 1]$ represents the proportion of sophisticated shareholders that sell their stock at the offer period and it is determined endogenously.

We assume that naïve shareholders follow the Board’s advice at $t = 2$. We further assume that the proportion of sophisticated shareholders, δ , is less than the proportion of control required by the bidder to allow her to benefit from the acquisition. This implies that the bidder cannot acquire sufficient control of the firm without the target Board’s recommendation. This implies that the takeover goes through if and only if (2.1) holds. This is broadly representative of takeovers of large companies, where both shareholder dispersion and the Board’s fiduciary duty (see footnote 9) yield the vast majority of takeover outcomes to be in line with the target Board’s recommendation. Several anecdotes suggest that bidders often only make a direct offer to target shareholders after the Board’s agreement, e.g. Kraft and Cadbury. More systematically, Baker and Savasoglu (2002) in their study of 1,901 US takeover offers between 1981 and 1996 report that in approximately 80% of cases, the outcome was in line with the Board’s advice.

Our assumption allows us to set aside the heavily studied shareholder coordination problem considered by Grossman and Hart (1980). Their work had shareholders hoping to hold on to shares to profit from a better run post-takeover firm. Our concern is rather shareholders seeking to determine the most profitable moment

to sell up. Sufficient naïve shareholders also sets aside the potential effects of communication frictions between shareholders and the Board studied in [Levit \(2017\)](#), and allow us to isolate the influence of early-sales on the final outcome.

If a shareholder decides to hold on to her shares through the offer period, then her payoff is determined at $t = 2$. If the Board elect to recommend the offer, then the deal is consummated and so the shareholder receives the offer price P . If instead the Board reject the offer then the deal fails and so the shareholder receives the stand-alone firm value: V .

Alternatively a shareholder may decide to sell her shares during the offer period, $t = 1$. In this case the shareholder receives the market price of the stock M . The bidder's offer price P is not available unilaterally to an individual shareholder in advance of the deal being consummated as the bidder's offer is conditional on enough shareholders accepting, and this will only be ascertained after the offer period (and so at $t = 2$). Hence $t = 1$ sales can only be at the market price. We detail this next.

2.2.4 Market price of a stock

We model the market in the stock as being provided by a competitive sector of market-makers. The market makers are constrained to make zero profits and so stand ready to buy shares at a market price M which allows them to break-even. This implies that the market price is given by:

$$M = \Pr[\text{Board accept offer}] \cdot P + \Pr[\text{Board reject offer}] \cdot V \quad (2.3)$$

We make two assumptions that are worth discussing. First, the market makers do not receive private signals as to the Board's type. They therefore calculate the expected probability of the deal being consummated as a function of the publicly known expected level of Board resistance, y , along with rational expectations of the behaviour of the shareholders in equilibrium. We believe the assumption is a good

approximation of reality in which shareholders of a stock who act strategically are likely to be sophisticated and have been exposed to the management of the firm over the duration of ownership of the stock. This experience may well allow the shareholders to ‘parse’ management actions more accurately and so give a more precise estimate of the Board’s level of resistance.

The second relevant assumption is that market makers do not observe the order flow, and set a price based on prior beliefs. Otherwise the absence of noise traders would allow market makers to perfectly infer the Board’s level of resistance θ and in turn the takeover outcome. As a result the game would yield multiple equilibria as described in the introduction. It is therefore possible to relax this assumption at the cost of introducing a proportion of shareholders acting as noise traders. This modification would reduce the sensitivity of the stock price to the strategic shareholder game without affecting our qualitative analysis. For the sake of simplicity we decide to exclude noise trade.

2.2.5 Game timing

For clarity we bring together the entire timeline of the model: At $t = 0$ the target company receives an offer P from the Bidder that everyone observes and θ is privately realised. The offer P is conditional on the bidder acquiring full control. During the interim $t = 1$, market makers set a price M which is a function of the probability that the takeover succeeds. Furthermore, each sophisticated shareholder i receives a private signal x_i , updates her beliefs about θ and then decides whether to sell or to hold her share. Finally, at $t = 2$ the Board decides whether to recommend for or against the offer after observing the proportion of shareholders who have sold during the interim period ρ , and enough shareholders follow the Board’s advice to ensure that the takeover only succeeds if it is recommended by the Board. We ignore temporal discounting between periods for the sake of simplicity. Figure 2.1 depicts the sequence of events.

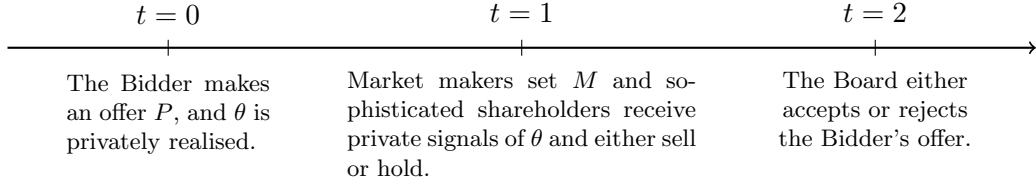


Figure 2.1: Timeline

2.2.6 Equilibrium concept

A strategy in our model is a mapping from the sophisticated shareholder's signal x_i to a binary decision to either sell or hold the stock during the offer period, $t = 1$. We look for a Bayesian Nash Equilibrium in pure strategies.

Definition 13 *An equilibrium of our model comprises a strategy for each sophisticated shareholder: $x_i \rightarrow \{\text{sell}, \text{hold}\}$ such that:*

1. *The Board agree to the bidder's offer if (2.1) holds;*
2. *The market makers set the interim market price of the stock according to (2.3);*
3. *The sophisticated shareholders' strategy constitutes a Bayesian Nash Equilibrium for each shareholder.*

We will look for Bayesian Nash (x^*, θ^*) –threshold equilibria, and show that one exists and it is unique.

Definition 14 *A Bayesian Nash (x^*, θ^*) –threshold equilibrium is one in which the sophisticated shareholders and the Board use threshold strategies. These require:*

1. *Shareholder i sells in the offer period if and only if her signal satisfies $x_i > x^*$;*
2. *The Board rejects the bidder's offer if and only if $\theta > \theta^*$.*

2.3 Equilibrium Solution

2.3.1 Existence and uniqueness of (x^*, θ^*) -threshold equilibrium

Suppose that sophisticated shareholders follow an x^* -threshold strategy: sell in the offer period if and only if they observe $x_i > x^*$. Then, for a given realisation of the Board's type θ , the proportion of sophisticated shareholders selling corresponds to the mass receiving signals above the threshold. Equivalently, $\gamma = \Pr[x > x^* | \theta]$. From (2.2) it follows that stock sales at the interim are

$$\rho = \delta [1 - \Phi(\sqrt{\tau_\varepsilon}(x^* - \theta))] \quad (2.4)$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal. We first show that in this setting the Board's decision rule is well-defined:

Lemma 15 *If sophisticated shareholders follow an x^* -threshold strategy, the Board has θ^* -threshold equilibrium behavior for $\tau_\varepsilon < 2\pi/(\kappa\delta)^2$. Further $\theta^*(x^*)$ is uniquely defined.*

Proof. See Technical Appendix. ■

If sophisticated shareholders follow an x^* -threshold strategy, the critical Board's type θ^* must satisfy

$$\theta^* = P - V + \kappa\delta [1 - \Phi(\sqrt{\tau_\varepsilon}(x^* - \theta^*))], \quad (2.5)$$

where we have substituted the volume of sales ρ in (2.4) into the Board's decision rule (2.1) evaluated at indifference. Lemma 15 confirms that equation (2.5) has a unique solution, and further that the Board will reject the bidder's offer for all $\theta > \theta^*$ and will accept it for all $\theta < \theta^*$. We pause here to note that the equilibrium exists if the shareholders' information is imprecise enough. This differs from standard global game analyses in which the agents' actions are complementary, and arises due to

the strategic substitutability of the shareholders' selling decision. We will discuss these information properties after deriving the equilibrium result in Proposition 17 below.

Now consider the market-makers. These set stock price M during the offer period based only on the public information y and knowledge of the equilibrium. Hence, the probability they assign to a takeover success is equal to the probability that θ lies below the critical θ^* conditional on the common prior $\theta \sim N(y, 1/\tau_\theta)$. We denote this probability β , so

$$\beta = \Pr[\theta \leq \theta^*] = \Phi(\sqrt{\tau_\theta}(\theta^* - y)). \quad (2.6)$$

Combining with (2.3) delivers the stock price during the offer period as:

$$M = \beta P + (1 - \beta) V = V + (P - V) \cdot \Phi(\sqrt{\tau_\theta}(\theta^* - y)) \quad (2.7)$$

We can now consider the selling strategy of a shareholder i receiving a signal x_i and facing the interim price in (2.7). She updates beliefs using Bayes' rule so that $\theta|x_i$ is normally distributed with mean $\frac{\tau_\theta y + \tau_\varepsilon x_i}{\tau_\theta + \tau_\varepsilon}$ and precision $\tau_\theta + \tau_\varepsilon$. Thus, if other shareholders follow an x^* -threshold strategy, then the Board will have a θ^* -threshold strategy (Lemma 15), and so the expected benefit to selling during the offer period over holding is:

$$\begin{aligned} u(x_i, x^*) &= M - (P - V) \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta y + \tau_\varepsilon x_i}{\tau_\theta + \tau_\varepsilon}\right)\right) - V \\ &= (P - V) \left[\Phi(\sqrt{\tau_\theta}(\theta^* - y)) - \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta y + \tau_\varepsilon x_i}{\tau_\theta + \tau_\varepsilon}\right)\right) \right] \end{aligned} \quad (2.8)$$

where we have substituted in the offer period stock price (2.7). Equation (2.8) shows that shareholder i 's value of selling is monotonically increasing in the signal received x_i . If $u(x^*, x^*) = 0$ has only one solution then we have demonstrated that there exists a unique Bayesian Nash x^* -threshold equilibrium. Lemma 16 formalises a

sufficient condition:

Lemma 16 *There exists an x^* -threshold equilibrium if $\tau_\varepsilon < 2\pi/(\kappa\delta)^2$, and further this equilibrium is unique.*

Proof. See Technical Appendix. ■

We are now in a position to solve our model explicitly:

Proposition 17 *Suppose that the precision of private noise satisfies $\tau_\varepsilon < 2\pi/(\kappa\delta)^2$, then there exists a unique Bayesian Nash (x^*, θ^*) -threshold equilibrium in which all shareholders sell their shares to the market if they observe a signal above x^* and hold them otherwise. The takeover succeeds if, and only if, the takeover resistance is below the threshold θ^* . The thresholds are characterised implicitly by the following equations:*

$$\theta^* = P - V + \kappa\delta\Phi(\Delta(\theta^* - y)) \quad (2.9)$$

$$x^* = \theta^* - \frac{\Delta}{\sqrt{\tau_\varepsilon}}(\theta^* - y) \quad (2.10)$$

where $\Delta \equiv \frac{\tau_\theta}{\sqrt{\tau_\varepsilon}} \left[\sqrt{1 + \frac{\tau_\varepsilon}{\tau_\theta}} - 1 \right]$.

Proof. Existence and uniqueness of the (x^*, θ^*) -threshold equilibrium is delivered by Lemmas 15 and 16. The explicit solution is solved for in the Technical Appendix.

■

The fundamental variables of the model $\{P - V, \kappa, \delta, y, \tau_\varepsilon, \tau_\theta\}$ determine the critical Board's type θ^* and in turn our main outcomes, namely the probability that the takeover succeeds (2.6), the interim stock price (2.7) and the stock sales at the offer period (2.4). To facilitate the subsequent analysis, we make use of expressions in Proposition 17 to characterise stock sales as a function of the Board's critical type:

$$\rho = \delta [\Phi(\Delta(\theta^* - y) - \sqrt{\tau_\varepsilon}(\theta^* - \theta))] \quad (2.11)$$

Note that our model is a global games analysis in which agents' actions are strategic substitutes in equilibrium, and not strategic complements as is often considered (Morris and Shin, 1998; Bebchuk and Goldstein, 2011). This follows as in an (x^*, θ^*) -threshold equilibrium the market makers will anticipate uncertainty as to whether the Board will accept or reject the bidder's offer, and so (2.3) implies that the firm's share price in the offer period will lie strictly between the pre-bid price of V and the offered price of $P > V$. The ideal outcome of each sophisticated shareholder is therefore to *hold* on to the shares during the offer period and receive P when the Board recommend acceptance, as long as enough other shareholders *sell* during the offer period to pressure the defensive Board into accepting the offer. This is a situation of strategic substitutes as if other shareholders were to reduce the volume of shares they sold during the offer period, then any given shareholder would wish to increase the volume of shares she sold in the offer period sufficiently to ensure the Board would still be compelled to agree to the bid.

2.3.2 Information Structure

The public and private information as to the Board's type is imperfect. The public, including the market makers, have an estimate of how defensive the Board is which has precision τ_θ . Sophisticated shareholders however have a further private signal of precision τ_ε . Beliefs are therefore dispersed. To better understand the effects of these assumptions, we study our equilibrium at limiting values.

Shareholders' information

First consider the limit in which private signals lose all precision ($\tau_\varepsilon \rightarrow 0$) and so are uninformative. In this case all information asymmetries disappear and sophisticated shareholders and market makers all share the same belief about the realisation of θ : the shareholders are therefore indifferent between trading and not. In this case alge-

braic manipulation confirms that $\Delta \rightarrow 0$ as $\tau_\varepsilon \rightarrow 0$.¹⁰ Equation (2.11) demonstrates that exactly half of the shareholders sell during the offer period in equilibrium. The randomisation arises via the dispersed uninformative private signal.

We have a unique equilibrium if the private information is not too informative: $\tau_\varepsilon < 2\pi/(\kappa\delta)^2$. At the opposite extreme of $\tau_\varepsilon \rightarrow 2\pi/(\kappa\delta)^2$ precision is held to finite levels and we will explore the behaviour of our model below. The uniqueness condition requiring private signals to not be too informative is in contrast to games of strategic complementarities, where uniqueness is guaranteed for sufficiently precise private information (Morris and Shin, 2003). In our strategic substitutes setting if agents can predict with high accuracy what signals others received, then the uniqueness of the pure strategy equilibrium breaks down: shareholders wish to mis-coordinate, selling (holding) if enough other shareholders hold (sell).

Public information

Now consider the limit in which public information loses all precision ($\tau_\theta \rightarrow 0$). In this case the market makers cannot refine their estimate of the probability the Board is sufficiently resistant to block the acquisition. Hence market makers will anticipate that the takeover has a 50% chance of success (equation 2.6). This implies that the price of the stock during the offer period lies equidistant between P and V . Shareholders will therefore wish to sell during the offer period if their private information leads them to believe the Board have a less than 50-50 chance of accepting the bid. This is why the critical shareholder threshold x^* is given by θ^* . This follows from equation (2.10) and the fact that algebraic manipulation yields that $\Delta \rightarrow 0$ as $\tau_\theta \rightarrow 0$.

¹⁰To see this, use L'Hôpital's rule and let $f(\tau_\varepsilon) = \tau_\theta \left[\sqrt{1 + \frac{\tau_\varepsilon}{\tau_\theta}} - 1 \right]$ and $g(\tau_\varepsilon) = \sqrt{\tau_\varepsilon}$. Then $\frac{f'(\tau_\varepsilon)}{g'(\tau_\varepsilon)} = \sqrt{\frac{\tau_\theta \tau_\varepsilon}{\tau_\theta + \tau_\varepsilon}}$ and $\lim_{\tau_\varepsilon \rightarrow 0} \frac{f'(\tau_\varepsilon)}{g'(\tau_\varepsilon)} = 0$.

2.3.3 Uniqueness

The condition $\kappa < \sqrt{2\pi}/\sqrt{\tau_\varepsilon}\delta$ is sufficient to guarantee existence and uniqueness of the equilibrium as defined in [Definition 14](#). Interpreted in this way our uniqueness condition limits the influence of new shareholders on the Board, or limits the vulnerability of the Directors to challenge of their business judgment. Thus, it restrains the magnitude of the externality of players' actions. This prevents situations where, for instance, a highly defensive Board incentivises a disproportionate volume of shareholders to sell during the offer period, increasing the internal pressure sufficiently so as to lead to a takeover success.

2.4 Model Analysis

We begin, in [Section 2.4.1](#) with a comparative statics analysis to understand the model solution; and we use this analysis to enrich the regulatory debate about the discretionary power of corporate Boards to influence against takeovers as described in the Introduction. We then use our model to study the marginal benefit to governments and outside bodies of pressurising Boards ([Section 2.4.2](#)). Finally we extend the model to endogenise the bid price in [Section 2.4.3](#).

2.4.1 Comparative Statics

We conduct comparative statics to study the reaction of the key outcomes in our model to changes in the main fundamentals: the Bidder's offer P , the influence of new shareholders κ , and the Board's expected level of resistance y . Below we investigate in turn the impact these fundamentals have on the probability of a takeover succeeding, the interim stock price, and the proportion of shareholders who sell-out before the Board make their recommendation.

Equilibrium response to changes in the bid premium

The bid premium in our model is given by $P - V$. We have:

- Proposition 18** 1. *The critical Board resistance threshold θ^* rises in the bid premium;*
2. *The critical shareholder signal threshold x^* rises in the bid premium;*
3. *The probability of the takeover succeeding rises in the bid premium;*
4. *The stock price during the offer period rises in the bid premium;*
5. *For a given level of Board resistance θ , the proportion of shareholders selling in the offer period falls in the bid premium.*

The Board is indifferent between accepting and rejecting the bidder's offer if equation (2.1) is satisfied with equality. If the bid premium were to rise then *ceteris paribus*, it is more likely that the Board will agree to the bid. This acts to raise the critical threshold θ^* above which the Board rejects the bid. As the Board is more likely to accept the bidder's offer, the critical threshold x^* above which shareholders decide to sell-up early is pushed up. That is the shareholders require a very unfavourable signal as to how defensive the Board is before they will decide to forgo some of the potential profits from the acquisition and sell during the offer period. Together we therefore have results 1 and 2.

The result that increasing the bid premium raises the probability of the takeover succeeding follows from the increase in the Board's threshold type. As the probability of a successful deal rises, the market makers raise the price of the stock during the offer period towards the bid price. The final result trades off two conflicting effects: on the one hand the probability the takeover will succeed going up implies that a shareholder sees more value in waiting for the Board to recommend the offer; on the other hand this raises the price during the offer period and so

increases the benefit of selling early. This tradeoff resolves itself against selling early for the sophisticated shareholders due to their informational advantage. Both the sophisticated shareholders and market makers appreciate that a higher bid premium raises the probability of the takeover being consummated. However the information available to the shareholders is more precise and so the sensitivity of the estimates of the probability of deal consummation to the Board's critical threshold is greater for shareholders. It therefore follows that as the Board's critical threshold θ^* rises, the market makers raise the expected probability of a deal, though not as much as shareholders do on average. And hence the price rise is not sufficient for many shareholders, and the volume of stock sales at the interim declines.

Equilibrium response to changes in the influence of shareholders' sales

The parameter κ accounts for the pressure that shareholders' sales during the offer period put on the Board to accept the takeover offer. We have:

- Proposition 19** 1. *The critical Board's resistance threshold θ^* rises in the influence of shareholders' sales during the offer period;*
2. *The critical shareholder signal threshold x^* rises in the influence of shareholders' sales;*
3. *The probability of the takeover succeeding rises in the influence of shareholders' sales;*
4. *The stock price during the offer period rises in the influence of shareholders' sales;*
5. *For a Board of resistance type θ , the proportion of shareholders selling in the offer period falls in the influence of shareholders' sales.*

The intuition behind Proposition 19 is analogous to that underlying Proposition 18. In both cases the model alteration (higher bid premium, or higher influence

of shareholders' sales) has the effect of, *ceteris paribus* making it more likely the bidder's offer will be accepted. In the prior Section 2.4.1 this was because of the increase in the bid premium; in this section it is because shareholders who sell have a stronger effect in destabilising the Board. The critical resistance threshold θ^* and the critical shareholder threshold x^* are therefore both pushed up making it less likely they will be triggered.

As above these effects conspire to raise both the probability the deal is consummated, and the price of the stock during the offer period. Once again this tension resolves itself against selling the stock. Shareholder turnover declines as the expectation of sophisticated shareholders for deal consummation is more sensitive to the shareholder power parameter κ than the market makers' due to their more precise information on the Board.

As noted in the introduction, a lively legal debate exists as to whether the business judgment rule should apply to takeover decisions and insulate the Directors from challenge. Different legal jurisdictions, even across different states in the US, differ in how much protection they offer Directors from allegations of breach of fiduciary duty. The protection afforded by the business judgment rule is captured in our model by the parameter κ . In particular, lower values of κ represent jurisdictions where Boards have more discretion and need pay less heed to the apparent contradiction of them arguing a takeover is bad whilst existing shareholders sell out at a price below that being offered. Our analysis suggests that empowering Board discretion in this way leads to a lower rate of takeover success, but perhaps surprisingly, it also leads to more shareholders voting to sell out. As the Board become more able to ignore shareholders selling in the offer period, the less likely the existing shareholders are to benefit from the bid price via Board deal acceptance, and so the more profitable it becomes to secure at least some of the gains by selling out early.

Shareholders' early sales also pressure the Board to accept the offer because

the acquirers are often risk arbitrageurs with a strong interest for the deal to be consummated. It has often been argued that these short-term investors may not have the best interests of the target firm at heart. Again, we explained in the Introduction that, policy makers, including from the US and UK, have sought to respond to this perceived problem by considering implementation of *disfranchisement* rules designed to ensure that the outcome of takeover bids is determined more by the core shareholder base, and less by short-term speculative investors.¹¹ This policy intervention is also captured by a reduction in κ , and so we again conclude that while disenfranchising new shareholders would indeed act to lower the probability of takeovers being successfully completed, a higher proportion of existing shareholders would decide to sell during the takeover struggle. Hence, although designed to help long-term shareholders by hurting short-term ones, this policy will not lead to greater long-term ownership in a takeover setting.

The predictions in Proposition 19 are analogous to those obtained when studying the equilibrium response to changes in the proportion of sophisticated shareholders δ . In our setting, an increase in the number of strategic shareholders is equivalent to increasing the impact and weight of their selling decision during the offer period. Thus if the proportion of sophisticated shareholders on the register were to rise, then the probability of the takeover succeeding would rise, as would the interim stock price. The effect on the volume of stock sales is however ambiguous as the probability of a given sophisticated shareholder selling declines, but the measure of sophisticated shareholders rises.

Equilibrium response to changes in expected level of takeover resistance

The parameter y is the prior expected level of the Board's resistance to accept the offer. It captures the ex ante expectation of market makers as well as the common

¹¹See for example: The Takeover Panel (2010), *Review of certain aspects of the regulation of takeover bids*, consultation paper issued by the Code Committee, London, 1 June 2010; and Department for Business & Innovation Skills (BIS) (2014), *Practical and legal issues related to limiting the rights of short-term shareholders during takeover bids*, Note of BIS roundtable, October 2014.

prior shared by all shareholders. We have:

- Proposition 20** 1. *The critical Board's resistance threshold θ^* falls in the prior expected level of resistance;*
2. *The critical shareholder signal threshold x^* rises in the prior expected level of resistance;*
3. *The probability of the takeover succeeding falls in the prior expected level of resistance;*
4. *The stock price during the offer period falls in the prior expected level of resistance;*
5. *For a Board of resistance type θ , the proportion of shareholders selling in the offer period falls in the prior expected level of resistance.*

Changes in the ex ante expectation market participants have as to the Board's type do not have a direct effect on the Board's decision: the parameter y is absent from the Board's decision condition (2.1). A change in the prior as to the Board's type therefore only has indirect effects: in this case through the critical threshold θ^* and the number of shareholders who sell up early ρ .

To disentangle the effects, suppose initially that the Board's critical threshold did not change in response to a small increase in the ex ante expected level of resistance, y . In this case as the prior expected level of the Board's resistance rises, so the probability the market makers assign to the deal being successfully consummated falls. In turn this has the effect of lowering the price of the stock during the offer period. If the critical Board threshold θ^* had not changed then the enhanced precision of the shareholders would not play a role in this thought experiment and so stock sales would fall (equation 2.11). However this in turn would weaken the pressure on the entrenched Board and so would itself make the

deal less likely. As a result pressure is created to lower the Board's critical threshold θ^* . This yields the first result.

The movement in the Board's critical threshold now creates opposing forces on the number of shareholders who exit the company at the interim ρ . We have already noted less shareholders sell their stock as the market makers lower its price in the offer period. However this alters the Board's critical threshold, and the shareholders' estimate of the change in the probability of acceptance is more sensitive due to their better information. Therefore sophisticated shareholders anticipate a larger reduction in the probability of deal acceptance than the market makers and so this acts to raise stock sales. The uniqueness condition which limits the influence of shareholders, or equivalently prevents private information being too precise, ensures that the first effect dominates: the shareholder turnover declines. To deliver this result it follows that the critical threshold at which the shareholders sell, x^* rises in the prior expected level of resistance, completing the intuition.

2.4.2 The marginal benefit of pressurising Boards

Examples of the British government exerting pressure on Boards to reject takeover offers include cases from the confectionery industry, as noted in the introduction, as well as the pharmaceutical, and oil industries.¹² Such political pressure is not confined to the UK, one can find reports in the popular press of governments encouraging firms to resist takeovers drawn perhaps from every major economy. Scholars have sought to clarify why governments should wish to oppose a free market in corporate control. For example [Roe \(1994\)](#) identifies a systematic alignment of the politicians' interests with those of corporate management. In the same spirit, [Hellwig \(2000\)](#) argues that the political system can be seen as a stakeholder in its own

¹²Press coverage includes: Bogdanor, A. (2014). The regulators were right to force Pfizer's hand. *Financial Times*, [online]. Available at: <http://www.ft.com/cms/s/0/a476fdf0-e5bc-11e3-a7f5-00144feabdc0.html#axzz3xbkC23LC> [Accessed 17 Jan. 2016]. See also Parker, G. and Christopher, A. (2015). UK government warns BP over potential takeover. *Financial Times*, [online]. Available at: <http://www.ft.com/cms/s/2/06a3207e-e901-11e4-87fe-00144feab7de.html#axzz3xbkC23LC> [Accessed 17 Jan. 2016]

right, while Jensen (1991) regards the treatment of corporate control by the political system as a reaction to populist rhetoric without an understanding of the systemic implications.

Here we do not take a stand on why government should wish to influence a Board's decision. Instead our focus is on understanding when political pressure is likely to be most effective, and so when it is most likely to occur. Thus this section considers the case of a government that seeks to pay a cost to alter y by exerting political pressure. We therefore hold that a government, or external pressure groups, cannot mandate exactly how resistant a Board should be; but they can encourage it to resist. Given the literature cited above we consider that the costs of opposing a takeover will most likely be incurred if the benefit, in terms of the reduced probability of the takeover being consummated, is sufficient. Hence we believe a government is most likely to pay the costs of intervention if $d\beta/dy$ is large and negative.

Our first main result on this theme is captured here:

Proposition 21 *$d\beta/dy$ is negative and monotonically decreasing in κ when public information is sufficiently noisy, i.e. small τ_θ .*

Proof. See Technical Appendix. ■

Proposition 21 implies that a government will have the greatest incentive to intervene to encourage a Board to ‘have the courage’ to reject a corporate takeover in economies in which managerial Boards are most heavily affected by shareholder sales. Thus, the government incentive to intervene when Boards are strongly protected by the business judgement rule will be weak. However, in jurisdictions in which the purchasers of new shares are very influential and have their shareholder rights respected, the incentive to intervene will be strong. This perhaps surprising result suggests that countries which pride themselves on respecting all shareholders' rights, perhaps such as the UK and the US, are the ones in which governments will have the greatest incentive to intervene by encouraging a Board to become entrenched

in the face of a politically undesirable takeover. However in the case of the US the extensive application of the business judgment rule might, in turn, lower the government incentive to intervene.

To understand this result note that if shareholder sales have a strong influence, that is κ is large, then the takeover success is highly sensitive to the actions of the existing shareholders. A small impact on the proportion of the shareholders selling during the offer period can therefore achieve a substantial reduction in the deal probability. In proposition 20 we noted that an increase in the prior expected level of takeover defence had the effect of increasing the critical threshold x^* for sophisticated shareholders and so lowering the proportion of shareholders who sell in the offer period. This lowers the pressure on the Board to agree the deal, and in economies with greater respect for shareholders (largest κ) the marginal change from this effect is the most pronounced. As a result the potential gains to government from intervention to alter expected takeover resistance y are at their greatest.

A second result is available:

Proposition 22 *$d\beta/dy$ is negative, quasiconvex in P , and reaches a minimum when $\beta = 1/2$.*

Proof. See Technical Appendix. ■

Proposition 22 implies that increases in political pressure exerted to endorse takeover defences have the greatest impact on the takeover's outcome when the probability of a success is otherwise close to a half. Hence, a government is most likely to find the cost of increasing political pressure in opposition to a bid worthwhile if the bidding offer is competitive, but not excessively so, thus implying that the success of the deal hangs in the balance. A graphical intuition is provided by Figure 2.2, where we plot $\frac{\partial\beta}{\partial y}$ as a function of P . The dashed line represents an arbitrary marginal cost of increasing y , and hence it delimits the corresponding range of takeover premiums for which the government is willing to exert political pressure.

It is possible to see that political pressure delivers the greatest impact when the bidder's offer is such that $\beta = \frac{1}{2}$.

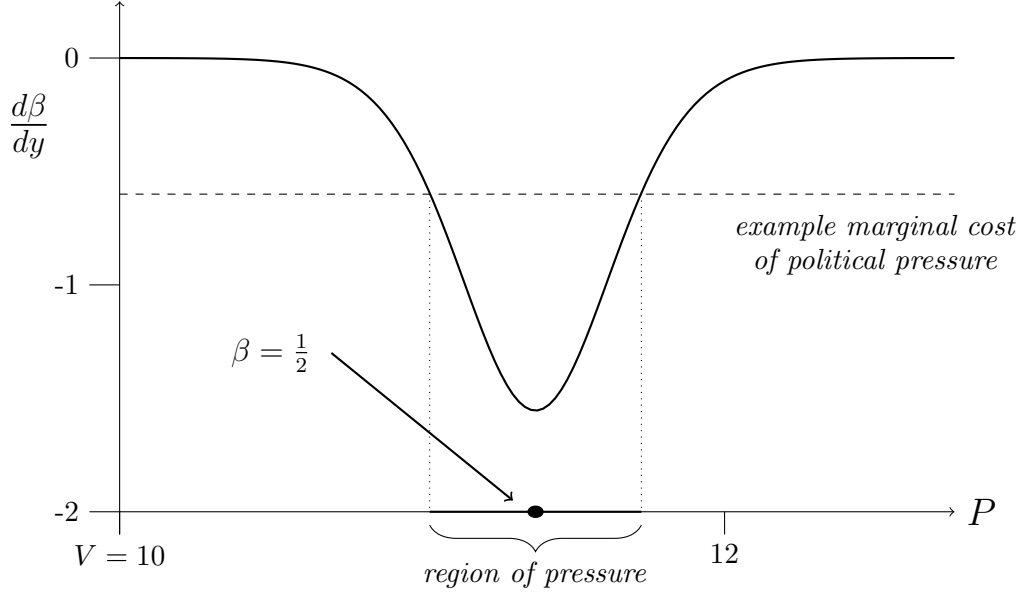


Figure 2.2: Numerical illustration of the effectiveness of increasing political pressure y as a function of the takeover bid P .

An arbitrary cost of increasing pressure is set at -0.8 . When $\frac{d\beta}{dy}$ overcomes this cost, the government increases pressure. The dot indicates $P = V + y - \frac{\kappa\delta}{2}$, which satisfies $\beta = \frac{1}{2}$. Parameter values are $V = 10$, $y = 2$, $\tau_\varepsilon = 4$, $\tau_\theta = 5$, $\kappa = 3.125$ and $\delta = 0.4$, and β is given in (2.6).

2.4.3 Endogenising the bid price

Thus far we have considered an exogenous takeover premium which we believe to be a good model of a contested takeover in which the bid price is forced above the level an individual bidder would bid in the absence of competing buyers. Nonetheless, numerous takeover offers are characterised by only one bidder.

This section studies the case of the takeover premium being strategically set by a Bidder with no competitors. Hence, the Bidder makes an offer maximizing his expected profit, which is the product of the surplus from the takeover and the probability that the takeover takes place. Suppose that the Bidder's value of the target company is $W > V$. The game proceeds as in the benchmark setting with the

only difference that, at $t = 0$, the offer P solves the following bidder optimisation problem:

$$\begin{aligned} & \max_{P \in [V, W]} \{ \Pi = (W - P) \beta(\theta^*(P)) \} \\ &= \max_{P \in [V, W]} \{ \Pi = (W - P) \Phi(\sqrt{\tau_\theta} [\theta^*(P) - y]) \} \end{aligned} \quad (2.12)$$

where $\theta^*(P)$ is implicitly characterised by equation (2.9). We assume that the bidding firm shares the same information set as the market makers – this prevents market participants seeking to make inferences as to the target Board’s entrenchment from the bidder’s bid price.¹³ We assume $P \in [V, W]$ for analytical simplicity and for realism. Nonetheless, in principle an offer $P < V$ can be optimal if the Board have expected entrenchment $y < 0$. This would be the case after political pressure urging a sale for example. We omit those cases where the target Board is keen to be replaced. The takeover offer determines both the Bidder’s surplus if the takeover succeeds and the probability that this occurs. The following Lemma characterises the solution to the Bidder’s problem:

Lemma 23 *Suppose the Bidder’s value for the target company is sufficiently large, i.e. $W - V > y - \frac{\kappa\delta}{2} + \sqrt{\pi/2\tau_\theta}$. Then, Π in (2.12) is quasiconcave and has an interior maximum at the optimal bid P^* which is implicitly characterised by:*

$$P^* = W - \frac{[1 - \kappa\delta\Delta\varphi(\Delta[\theta^*(P^*) - y])]\Phi(\sqrt{\tau_\theta}[\theta^*(P^*) - y])}{\sqrt{\tau_\theta}\varphi(\sqrt{\tau_\theta}[\theta^*(P^*) - y])} \quad (2.13)$$

Proof. See Technical Appendix. ■

For analytic solutions we restrict to the case in which the surplus available is large enough. This allows us to show that the Bidder’s profit function is quasiconcave. The Lemma implies that the optimal bid always satisfies $\theta^*(P^*) > y$ and

¹³Other papers in the literature of takeovers have assumed that shareholders have better information than the Bidder. [Ekmekci and Kos \(2016\)](#) argue that this can be thought of as a reduced form of a model in which the Bidder has some information, yet this information is public.

therefore, the probability that the takeover succeeds conditional on an offer being made is above one half.

This is a complicated setting which makes analysis difficult. Nonetheless we can establish a number of comparative statics results with respect to the expected level of the Board's resistance, y .

Proposition 24 *With a strategic Bidder:*

1. *The equilibrium bid price P is increasing in the prior expected level of takeover resistance;*
2. *The probability the takeover succeeds falls in the prior expected level of resistance;*
3. *The stock price during the offer period is affected in an ambiguous direction by the prior expected level of takeover resistance.*

Proof. The proofs of parts 1 and 2 are contained in the Technical Appendix. The proof of part 3 is by virtue of the numerical example in Figure 2.3. ■

The Proposition indicates the Bidder responds to increases in the prior expected level of takeover resistance with a bigger bid premium. This increase in the offer is designed to counter the expected takeover defence and to do enough to encourage the sophisticated shareholders to pressure the Board to agree to the bid. Nonetheless, the positive effect of a higher bid on the probability of a takeover is ultimately insufficient to outweigh the negative impact of a sufficiently great level of Board resistance. Hence the takeover becomes less likely.

Figure 2.3 plots both the endogenous takeover bid P^* and the interim stock price M as a function of the Board's expected type y for a given numerical example. It is possible to see that the takeover premium increases monotonically in response to a stronger expected defence, conforming to result 1 of Proposition 24. It is also possible to see that the price of the stock in the offer period, M , presents an

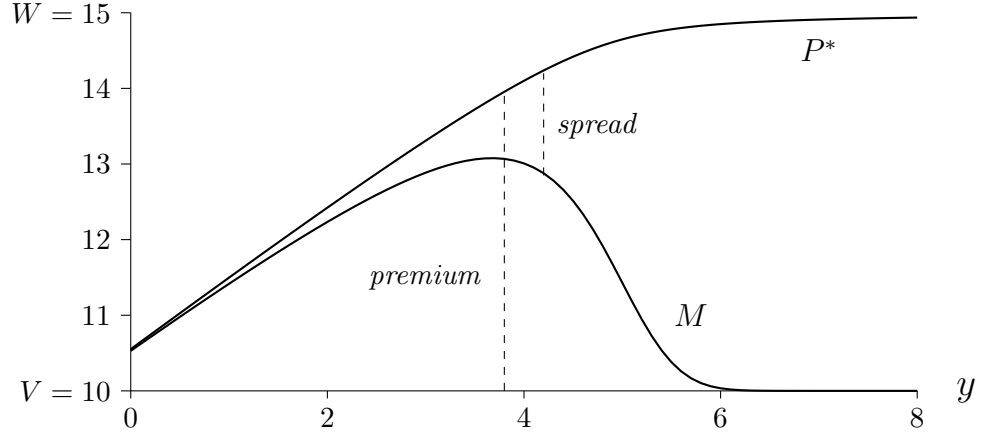


Figure 2.3: Optimal takeover bid P^* and the corresponding interim period stock price M as a function of takeover resistance y .

The takeover premium is $P^* - V$ whereas the spread from which risk-arbitrageurs aim to make profit is $P^* - M$. Parameter values are $W = 15$, $V = 10$, $\tau_\varepsilon = 4$, $\tau_\theta = 5$, $\kappa = .75$ and $\delta = .4$.

inverted-U shape relationship with the expected takeover resistance. As we consider ever higher levels of the Board's expected resistance, it is not possible for the increase in the bid price to rise sufficiently to maintain the probability of takeover success. As a result the probability of offer acceptance eventually falls and so the price of the stock during the offer period ultimately collapses.

2.5 Empirical predictions and analysis robustness

2.5.1 Empirical predictions

Our analysis offers a number of testable empirical predictions. Some of the most salient predictions concern the stock sales during the offer period or, equivalently, shareholder turnover, which get to the heart of the interrelationship between the Board and the shareholders. Our results also have testable implications relating Board entrenchment and takeover outcomes. The predictions we would highlight are:

1. The proportion of shareholders selling during the offer period falls in the bid

premium.

2. On average a greater proportion of shareholders sell during the offer period where managerial Boards have more discretionary power to resist takeovers.
3. Bids from uncontested acquisitions for firms with more resistant Boards are higher on average, and yet have lower completion rates.

The link between the bid premium and stock sales during the offer period is delivered by result 5 of Proposition 18. The prediction that regimes where managerial Boards enjoy great discretionary power to oppose takeovers, that is regimes with a low κ , should see more selling during the offer period is delivered by result 5 of Proposition 19. Both of these predictions concern the likely order flow during the offer period. Data exists on order flows and therefore the first prediction is potentially practically testable, though we are not aware of a test of the hypothesis existing. The second test involves comparisons across jurisdictions whilst controlling for shareholder rights versus the discretionary power of managerial Boards. Alternatively one could also compare takeovers across companies with different corporate charters within the same jurisdiction. Once again we are not aware of existing tests of the hypothesis. Constructing measures of shareholder rights is challenging; though La Porta et al. (1997) offer a blueprint. This second test is therefore approachable in principle.

The third prediction concerning the expected takeover resistance on acquisition premia, and on the probability of success, is delivered by results 1 and 2 in Proposition 24. A natural way to implement a test is to posit that entrenched Boards can extract greater rents from their firm in terms of high pay. Bebchuk et al. (2009) argue that this is a valid inference to make. The prediction would therefore be a joint hypothesis that uncontested (and therefore strategically priced) bids for firms where managers are overpaid will be bids for firms whose managers are entrenched, and so will result in higher offer premia on average and yet succeed less

frequently. Our prediction is in line with evidence of a relation between managerial entrenchment and the probability of deal failure (e.g. [Bebchuk et al., 2002](#)); though it is in contrast to results that find no significant effects (e.g. [Heron and Lie, 2006](#); [Bates et al., 2008](#)). Consistent with the hypothesis that Boards resist in the interest of shareholders, [Comment and Schwert \(1995\)](#) and [Bates and Becher \(2016\)](#) find that takeover defences increase bidding premia, as we predict.

2.5.2 Alternative Settings and Robustness

This section considers the robustness of our results to two variations of the benchmark model.

Cross-Ownership. In some cases target shareholders own stock of the bidding firm. If the takeover succeeds, these shareholders receive an additional payoff (either positive or negative) that accounts for the effect of the takeover on the bidder’s stock. [Appendix 2.8.1](#) studies this setting. We show that despite the change in the payoff function, the strategic interaction between shareholders and so our analysis is not materially altered. Intuitively this follows as each shareholder’s buy or sell decision is negligible on the probability of takeover success as each shareholder is assumed to be small; thus the strategic decision on whether to buy or sell can be divorced from changes in the value of the holding in the bidder. As a consequence [Proposition 17](#) holds, and hence our results continue to hold as stated and are entirely robust to portfolios of ownership.

Informed External Investors. We assume that sophisticated shareholders are better informed about the firm they own than other investors. As noted above this assumption seems to us natural as sophisticated shareholders, having been owners of the stock for a period of time, will have developed some knowledge of the preferences and practices of management. This assumption has the implication that sophisticated shareholders have positive expected returns. However, as noted in the Introduction, the acquirers of the stock during the offer period are often risk

arbitrageurs or hedge fund activists which are also likely informed and appear to make positive profits (see [Mitchell and Pulvino, 2001](#) and references therein).

[Appendix 2.8.2](#) considers an alternative information structure privileging external investors. In this appendix we assume that the potential buyers of stock receive private signals about the Board's type. In turn we assume that existing shareholders and market makers can only act on the common prior. It therefore follows that current shareholders will buy or sell in the offer period purely on whether the external investors are willing to pay above the ex ante expected price for the stock based on publicly available information. That is outside investors face completely elastic supply at the market price M .

We solve this alternative formulation in [Appendix 2.8.2](#). This setting is one of strategic complementarities for external investors, as their expected payoff from acquiring stock increases with the stock owned by other investors.¹⁴ Despite the new information structure, both the probability of a takeover and the interim stock value react to changes in the fundamentals as in Propositions [18](#), [19](#), and [20](#). However the relationship between the stock sales during the offer period and the fundamentals is reversed. If external investors have better information than current owners of the stock then their inferences when the Board's threshold type changes are more sensitive than those of market makers and existing shareholders. For example, if the bid price rises, this acts to raise the Board's critical threshold and make the takeover more likely to succeed. The market price therefore rises, but further profits can be made by the party with the better information. If this party is the external investors then they buy more of the stock and so lead to an increase in the shareholder turnover during the offer period, not a reduction as in Proposition [18](#).

Whether our benchmark formulation giving an information advantage to ex-

¹⁴The complementarities between external investors are similar, in spirit, to those studied in [Brav et al. \(2016\)](#), which represents the cost of engaging entrenched management to increase firm value. Our paper shows that in this situation, the game of strategic complementarities for stock acquirers is a game of strategic substitutes for sellers.

isting sophisticated shareholders is appropriate is ultimately an empirical question. As we noted above the empirical evidence on the relationship between the order flow in the offer period and the fundamentals we study is still in its infancy, and so the most appropriate formulation remains a matter of conjecture.

2.6 Conclusion

This study combines the following three observations into a formal analysis of takeovers: firstly managers often oppose takeovers, either due to the private benefits of corporate control, or because they consider the takeover offer to undervalue the company; secondly politicians often use pressure on Boards as a tool of industrial policy; and finally shareholder actions form a key part of the strategic creation of pressure on Boards and so complete a feedback loop between current shareholder actions and corporate decisions. Shareholders therefore can sway a Board to accept an offer – as happened in the case-study of the Kraft-Cadbury merger described in the Introduction.

Shareholders in our analysis therefore face a novel coordination problem: during the offer period, and so in advance of the Board having declared its position, sophisticated shareholders have to decide whether to hold their stock in the hope that the takeover goes through or to sell out early to partially profit from the elevated market price of the stock. Selling out raises the pressure on Boards to agree the takeover; both because they are indicative of a loss of confidence in the independent life of the firm, and also because new purchasers are almost certain to vote for acceptance. As a consequence, each shareholder wants to sell her stock when other shareholders hold it, but to hold it when other shareholders sell. Hence, selling becomes a public good to which no one wants to contribute. In equilibrium, selling decisions and takeover success are determined by the bid premium, the external pressure against the takeover and the influence of new shareholders.

Solving this model of strategic substitutes in a global games setting presents novel problems, however a unique threshold equilibrium can be shown to exist and can be characterised. This allows us to study and draw insights from a model with a unique pure strategy equilibrium which is not possible without the global games machinery. The model allows us to develop a unified analysis as to how outcomes of interest are affected by fundamentals. This allows us to disentangle two competing effects: increases in the probability a deal will succeed reduce the incentive for shareholders to sell early which in turn lowers the probability the deal will succeed. These opposing forces nonetheless are brought into equilibrium; and the requirements on the informational structure for the analysis to be well defined deliver a unique resolution of this tension. Thus the probability of a takeover succeeding rises in both the bid premium and the respect for shareholders – even though both of these make selling in the offer period less attractive and so in part lower the pressure on the Board to agree a deal.

Our work provides an entirely new analysis of two policy areas which, in different ways, play critical roles in takeovers: the discretionary power of managerial Boards; and external, often political, pressure. Jurisdictions differ on the power that managerial Boards have to recommend against a takeover. Perhaps unexpectedly, empowering Boards to avoid takeover outcomes being driven by market speculation during the offer period increases the number of shareholders that sell out during the offer period to profit from the increased price. Relatedly, policy makers have sought to bolster long term ownership by inhibiting the voting rights of short-term owners of a stock. This has the negative consequence of existing shareholders selling out in greater volumes during the offer period, which is the exact reverse of the outcome hoped for in the case of Cadbury by the outgoing chairman. Policy makers have also often sought to encourage Boards to reject politically unwanted takeovers. We see that this behaviour is most likely, *ceteris paribus*, in jurisdictions with the greatest respect for shareholder rights. The marginal benefit to political pressure, in terms

of reduction in the probability a takeover is successful, is greatest in countries which perhaps pride themselves on the fairness of their shareholder rules.

This analysis highlights the strategic importance of shareholder sales during the offer period, however the empirical predictions generated run ahead of the curve of empirical work. Though case studies are supportive we look forward to a more systematic empirical treatment of the themes we have explored.

2.7 Appendix: Proofs

2.7.1 Proof of Proposition 15

The model requires the Board to refuse to sell if and only if $\theta \geq P - V + \kappa\rho$. Substituting for $\rho = \delta\gamma$ using (2.4), define the function:

$$X(\theta) \triangleq \theta + \kappa\delta\Phi(\sqrt{\tau_\varepsilon}(x^* - \theta)) - [P - V + \kappa\delta]. \quad (2.14)$$

$X(\theta)$ is the Board's benefit to rejecting the offer over acquiescing. The Board rejects the offer if and only if $X(\theta) \geq 0$. The lemma follows if $X(\theta^*) = 0$ has a unique solution and $X(\theta)$ is increasing.

If $\theta > [P - V + \kappa\delta]$ then $X(\theta) > 0$, if $\theta < P - V$ then $X(\theta) < 0$, and so by continuity $X(\theta)$ has at least one root. Observe $X'(\theta) = 1 - \kappa\delta\sqrt{\tau_\varepsilon}\varphi(\sqrt{\tau_\varepsilon}(x^* - \theta))$. Given that $\max \varphi(\cdot) = \varphi(0) = 1/\sqrt{2\pi}$ the result follows. ■

2.7.2 Proof of Lemma 16

As $\frac{d}{dx_i}u(x_i, x^*) > 0$, there is a unique x^* -threshold equilibrium if $u(x^*, x^*) = 0$ has a unique solution. Observe that $u(x^*, x^*) < 0$ for $x^* \leq y$, and $\lim_{x^* \rightarrow \infty} u(x^*, x^*) > 0$, hence at least one root exists by continuity and the intermediate value theorem. We now wish to show at most one solution to $u(x^*, x^*) = 0$ can exist. Using (2.8) at all such solutions we have

$$\begin{aligned} \Phi(\sqrt{\tau_\theta}(\theta^* - y)) &= \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta y + \tau_\varepsilon x^*}{\tau_\theta + \tau_\varepsilon}\right)\right) \\ &\Rightarrow \sqrt{\tau_\theta}(\theta^* - y) = \sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta y + \tau_\varepsilon x^*}{\tau_\theta + \tau_\varepsilon}\right) \end{aligned}$$

where θ^* is a function of x^* and is given in (2.5). Hence define the function

$$Y(x^*) \triangleq \sqrt{\tau_\theta}(\theta^* - y) - \sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta y + \tau_\varepsilon x^*}{\tau_\theta + \tau_\varepsilon}\right) \quad (2.15)$$

As $\theta^* \in [P - V, P - V + \kappa]$, and so is bounded, $Y(x^*) = 0$ has at least one solution using the intermediate value theorem. Note $Y'(x^*) > 0$ if $\frac{d\theta^*}{dx^*} < 0$. The latter follows given $\kappa < \frac{\sqrt{2\pi}}{\delta\sqrt{\tau_\varepsilon}}$. Hence there exists only one solution to $Y(x^*) = 0$. The result follows. ■

2.7.3 Proof of Proposition 17

Existence and uniqueness of the equilibrium is delivered by Lemmas 15 and 16. Using the function in (2.15), observe that $Y(x^*) = 0$ satisfies

$$x^* - \theta^* = -\frac{\tau_\theta}{\tau_\varepsilon} \left[\sqrt{1 + \frac{\tau_\varepsilon}{\tau_\theta}} - 1 \right] (\theta^* - y)$$

The signal threshold in (2.10) follows immediately. The same condition can be combined with (2.5) to obtain the critical Board type in (2.9) using the fact that $\Phi(x) = 1 - \Phi(-x)$. ■

2.7.4 Proof of Propositions 18–20

We first establish the relationship between the critical boundaries (θ^*, x^*) with respect to the fundamentals in two lemmas. We then establish the remaining properties in a series of results.

Lemma 25 *The threshold θ^* is positively related to the Bid premium, the extent of new shareholders' pressure and the proportion of sophisticated shareholders, and negatively related to the Board's expected type, i.e. $\frac{\partial \theta^*}{\partial (P-V)} > 0$, $\frac{\partial \theta^*}{\partial \kappa} > 0$, $\frac{\partial \theta^*}{\partial \delta} > 0$ and $\frac{\partial \theta^*}{\partial y} < 0$.*

Proof.

1. *Bid Premium.* Define $F(\theta^*, P - V) = \theta^* - \text{RHS}(2.9)$ and use the Implicit

Function Theorem (IFT) so that $\frac{\partial \theta^*}{\partial (P-V)} = -\frac{\partial F(\theta^*, P-V)/\partial (P-V)}{\partial F(\theta^*, P-V)/\partial \theta^*}$. Then,

$$\frac{\partial \theta^*}{\partial (P-V)} = \frac{1}{1 - \kappa \delta \Delta \varphi(\Delta [\theta^* - y])} > 0 \quad (2.16)$$

under uniqueness. To derive the sign of the denominator note that we can guarantee that it is positive if $\kappa < \sqrt{2\pi}/\delta \Delta$, as $\varphi(\cdot)$ takes maximum value at $1/\sqrt{2\pi}$. Then, since $\sqrt{\tau_\varepsilon} > \Delta$ the uniqueness condition is sufficient.

2. *Internal Pressure.* Use the IFT so that $\frac{d\theta^*}{d\kappa} = -\frac{\partial F(\theta^*, \kappa)/\partial \kappa}{\partial F(\theta^*, \kappa)/\partial \theta^*}$. Then,

$$\frac{d\theta^*}{d\kappa} = \frac{\delta \Phi(\Delta [\theta^* - y])}{1 - \kappa \delta \Delta \varphi(\Delta [\theta^* - y])} > 0 \quad (2.17)$$

3. *Sophisticated shareholders.* Use the IFT so that $\frac{d\theta^*}{d\delta} = -\frac{\partial F(\theta^*, \delta)/\partial \delta}{\partial F(\theta^*, \delta)/\partial \theta^*}$. Then,

$$\frac{d\theta^*}{d\delta} = \frac{\kappa \Phi(\Delta [\theta^* - y])}{1 - \kappa \delta \Delta \varphi(\Delta [\theta^* - y])} > 0 \quad (2.18)$$

4. *Board's Expected Type.* Use the IFT so that $\frac{\partial \theta^*}{\partial y} = -\frac{\partial F(\theta^*, y)/\partial y}{\partial F(\theta^*, y)/\partial \theta^*}$. Then,

$$\frac{\partial \theta^*}{\partial y} = -\frac{\kappa \delta \Delta \varphi(\Delta [\theta^* - y])}{1 - \kappa \delta \Delta \varphi(\Delta [\theta^* - y])} < 0 \quad (2.19)$$

■

Lemma 26 *The threshold x^* is positively related to the Bid premium, the extent of new shareholders' pressure, the proportion of sophisticated shareholders and the Board's expected type, i.e. $\frac{\partial x^*}{\partial (P-V)} > 0$, $\frac{\partial x^*}{\partial \kappa} > 0$, $\frac{\partial x^*}{\partial \delta} > 0$ and $\frac{\partial x^*}{\partial y} > 0$.*

Proof. The sign of the partial derivatives with respect to $P - V$, κ , and δ , follow from implicit differentiation of (2.10) combined with the observation that $\sqrt{\tau_\varepsilon} > \Delta$. For the final result note that $\frac{dx^*}{dy} = \frac{\partial x^*}{\partial y} + \frac{\partial x^*}{\partial \theta^*} \frac{\partial \theta^*}{\partial y}$, where $\frac{\partial \theta^*}{\partial y}$ is characterized by (2.19). It follows that the uniqueness condition $\kappa < \frac{\sqrt{2\pi}}{\delta \sqrt{\tau_\varepsilon}}$ guarantees $\frac{dx^*}{dy} > 0$. ■

We then have:

- (a) *Bid Premium*. Note in (2.6) that $\frac{d\beta}{d(P-V)} = \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial(P-V)}$ where $\frac{\partial\beta}{\partial\theta^*} > 0$ and $\frac{\partial\theta^*}{\partial(P-V)} > 0$ - see (2.16). Furthermore, $\frac{dM}{d(P-V)} = \frac{\partial M}{\partial(P-V)} + \frac{\partial M}{\partial\beta} \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial(P-V)}$ with $\frac{\partial M}{\partial(P-V)} > 0$ and $\frac{\partial M}{\partial\beta} > 0$. Finally, from (2.11), $\frac{d\rho}{d(P-V)} = \frac{\partial\rho}{\partial\theta^*} \frac{\partial\theta^*}{\partial(P-V)}$ where $\frac{\partial\rho}{\partial\theta^*} < 0$ because $\sqrt{\tau_\varepsilon} - \Delta > 0$.
- (b) *Internal pressure*. First, notice in (2.6) that $\frac{d\beta}{d\kappa} = \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial\kappa}$ where $\frac{\partial\beta}{\partial\theta^*} > 0$ and $\frac{\partial\theta^*}{\partial\kappa} > 0$ - see (2.17). Moreover, $\frac{dM}{d\kappa} = \frac{\partial M}{\partial\beta} \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial\kappa} > 0$ since $\frac{\partial M}{\partial\beta} > 0$. Last, $\frac{d\rho}{d\kappa} = \frac{\partial\rho}{\partial\theta^*} \frac{\partial\theta^*}{\partial\kappa} < 0$ because $\frac{\partial\rho}{\partial\theta^*} < 0$.
- (c) *Sophisticated shareholders*. From (2.6) it follows that $\frac{d\beta}{d\delta} = \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial\delta}$ where $\frac{\partial\beta}{\partial\theta^*} > 0$ and $\frac{\partial\theta^*}{\partial\delta} > 0$ - see (2.18). Further, $\frac{dM}{d\delta} = \frac{\partial M}{\partial\beta} \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial\delta} > 0$.
- (d) *Board's Expected Type*. The effect on β is $\frac{d\beta}{dy} = \frac{\partial\beta}{\partial y} + \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial y} < 0$ because $\frac{\partial\beta}{\partial y} < 0$; $\frac{\partial\beta}{\partial\theta^*} > 0$ and $\frac{\partial\theta^*}{\partial y} < 0$. Furthermore, $\frac{dM}{dy} = \frac{\partial M}{\partial\beta} \frac{\partial\beta}{\partial\theta^*} \frac{\partial\theta^*}{\partial y} < 0$ because $\frac{\partial M}{\partial\beta} > 0$. Finally, note in (2.11) that

$$\begin{aligned}
\frac{d\rho}{dy} &\stackrel{sign}{=} \Delta \left(\frac{\partial\theta^*}{\partial y} - 1 \right) - \sqrt{\tau_\varepsilon} \frac{\partial\theta^*}{\partial y} \\
&= \sqrt{\tau_\varepsilon} \left(\frac{\kappa\delta\Delta\varphi(\Delta[\theta^* - y])}{1 - \kappa\delta\Delta\varphi(\Delta[\theta^* - y])} \right) - \Delta \left(\frac{\kappa\delta\Delta\varphi(\Delta[\theta^* - y])}{1 - \kappa\delta\Delta\varphi(\Delta[\theta^* - y])} + 1 \right) \\
&= [\sqrt{\tau_\varepsilon}\kappa\delta\varphi(\Delta[\theta^* - y]) - 1] \left(\frac{\Delta}{1 - \kappa\delta\Delta\varphi(\Delta[\theta^* - y])} \right) < 0
\end{aligned}$$

Where the last inequality follows from the fact that $\sqrt{\tau_\varepsilon}\kappa\delta\varphi(\Delta[\theta^* - y]) < 1$ under the uniqueness condition.

■

2.7.5 Proof of Proposition 21

Notice that

$$\begin{aligned}\frac{d^2\beta}{d\kappa dy} &= \frac{d}{d\kappa} \left(\frac{d\beta}{dy} \right) = \frac{d}{d\kappa} \left(-\frac{\sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta} (\theta^* - y))}{1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))} \right) \\ &= \text{sign} - \left[\left(\frac{\partial \theta^*}{\partial \kappa} \right) \tau_\theta \varphi'(\sqrt{\tau_\theta} (\theta^* - y)) \right] [1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))] \\ &\quad - [\sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta} (\theta^* - y))] \left[\delta \Delta \varphi(\Delta (\theta^* - y)) + \left(\frac{\partial \theta^*}{\partial \kappa} \right) \kappa \delta \Delta^2 \varphi'(\Delta (\theta^* - y)) \right]\end{aligned}$$

where $\frac{\partial \theta^*}{\partial \kappa} > 0$ by Lemma 25. Furthermore, note both $\varphi'(\sqrt{\tau_\theta} (\theta^* - y)) > 0$ and $\varphi'(\Delta (\theta^* - y)) > 0$ if and only if $\theta^* < y$. As a result, $\frac{\partial^2 \beta}{\partial \kappa \partial y} < 0$ for $\theta^* \leq y$ or equivalently, for $\kappa \leq \frac{2}{\delta} [y - (P - V)]$.

Consider $\kappa \in \left(\frac{2}{\delta} [y - (P - V)], \frac{\sqrt{2\pi}}{\delta \sqrt{\tau_\epsilon}} \right)$ so that $\theta^* > y$. Using that $\varphi'(x) = -x\varphi(x)$ we have

$$\frac{d^2\beta}{d\kappa dy} = \text{sign} \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta} (\theta^* - y)) \left\{ \begin{aligned} &(\theta^* - y) \left(\frac{\partial \theta^*}{\partial \kappa} \right) \tau_\theta [1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))] \\ &+ (\theta^* - y) \left(\frac{\partial \theta^*}{\partial \kappa} \right) \kappa \delta \Delta^3 \varphi(\Delta (\theta^* - y)) \\ &- \delta \Delta \varphi(\Delta (\theta^* - y)) \end{aligned} \right\}$$

Now using the fact that $\left(\frac{\partial \theta^*}{\partial \kappa} \right) [1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))] = \delta \Phi(\Delta (\theta^* - y))$ we can write

$$\frac{d^2\beta}{d\kappa dy} = \text{sign} \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta} (\theta^* - y)) \delta \Delta \varphi(\Delta (\theta^* - y)) \left\{ (\theta^* - y) \left[\frac{\tau_\theta \Phi(\Delta (\theta^* - y))}{\Delta \varphi(\Delta (\theta^* - y))} + \left(\frac{\partial \theta^*}{\partial \kappa} \right) \kappa \Delta^2 \right] - 1 \right\}$$

As $\theta^* < P - V + \delta \kappa$ we can guarantee that the brace is negative if

$$(\theta^* - y) \left[\frac{\tau_\theta \Phi(\Delta (\theta^* - y))}{\Delta \varphi(\Delta (\theta^* - y))} + \frac{\delta \Phi(\Delta (\theta^* - y))}{[1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))]} \kappa \Delta^2 \right] < 1$$

We know that this is true if τ_θ is small enough as $\lim_{\tau_\theta \rightarrow 0} \tau_\theta / \Delta = 0$ and $\lim_{\tau_\theta \rightarrow 0} \Delta =$

0. ■

2.7.6 Proof of Lemma 22

The relation between θ^* and y is crucial to characterise the effect of fundamentals. In expression (2.9) it is possible to see that $\theta^* = y \iff y = P - V + \frac{\kappa\delta}{2}$. Hence, $\frac{\partial\theta^*}{\partial y} < 0$ from Lemma 4 implies that $\theta^* > y \iff y < P - V + \frac{\kappa\delta}{2}$. Furthermore, notice in (2.6) that $\beta > \frac{1}{2} \iff y < P - V + \frac{\kappa\delta}{2}$. We use these results as a reference in our analysis.

Note that

$$\frac{d^2\beta}{dPdy} = \frac{\partial}{\partial y} \left(\frac{\sqrt{\tau_\theta}\varphi(\sqrt{\tau_\theta}(\theta^* - y))}{1 - \kappa\delta\Delta\varphi(\Delta(\theta^* - y))} \right)$$

Therefore $\frac{d^2\beta}{dPdy} > 0 \iff y < P - V + \frac{\kappa\delta}{2}$ or equivalently, $\frac{d^2\beta}{dPdy} > 0 \iff P - V > y - \frac{\kappa\delta}{2}$.

Hence, $\frac{d\beta}{dy}$ is quasiconvex in $P - V$ with a minimum at $P - V = y - \frac{\kappa\delta}{2}$. ■

2.7.7 Proof of Lemma 23

As we are maximising a continuous function over a compact set, a solution exists. We have that the optimal bid $P^* < W$ as $[d\Pi/dP]_{P=W} = -\beta(\theta^*(P)) < 0$. Now note that

$$\begin{aligned} \frac{\partial\Pi}{\partial P} &= -\Phi(\sqrt{\tau_\theta}[\theta^*(P) - y]) + (W - P)\sqrt{\tau_\theta}\frac{\partial\theta^*(P)}{\partial P}\varphi(\sqrt{\tau_\theta}[\theta^*(P) - y]) \quad (2.20) \\ &= \left[\frac{(W - P)\sqrt{\tau_\theta}}{1 - \kappa\delta\Delta\varphi(\Delta[\theta^*(P) - y])} - \frac{\Phi(\sqrt{\tau_\theta}[\theta^*(P) - y])}{\varphi(\sqrt{\tau_\theta}[\theta^*(P) - y])} \right] \varphi(\sqrt{\tau_\theta}[\theta^*(P) - y]) \end{aligned}$$

We can guarantee that a solution satisfies $P^* \geq V + y - \frac{\kappa\delta}{2}$, if $\frac{\partial\Pi}{\partial P} > 0$ for $P < V + y - \frac{\kappa\delta}{2}$. Next, we show that this holds for $P < W - \sqrt{\pi/2\tau_\theta}$. Use the following claims:

- **Claim 27** $\theta^*(P) < y$ and $\beta(\theta^*(P)) < \frac{1}{2}$ if and only if $P < V + y - \frac{\kappa\delta}{2}$.

Proof of Claim 27: Note that $\frac{\partial\theta^*(P,y)}{\partial y} < 0$ and $\theta^*(P, y) = y$ when $P = y + V - \frac{\kappa\delta}{2}$

- **Claim 28** $\frac{\Phi(x)}{\varphi(x)}$ is increasing in x . Hence, $\max_x \left\{ \frac{\Phi(x)}{\varphi(x)} \right\} |_{x \leq 0} = \sqrt{\frac{\pi}{2}}$.

Proof of Claim 28: The result is immediate for $x \geq 0$. Consider $x < 0$. The result follows by differentiation if $\varphi^2(x) - \Phi(x) \varphi'(x) > 0$. As $\varphi'(x) = -x\varphi(x)$ we wish to show $\varphi(x) + x\Phi(x) > 0$. Set $y = -x > 0$, and using symmetry we require $\varphi(y) - y(1 - \Phi(y)) > 0$ for $y > 0$. This follows if $(1 - \Phi(y)) / \varphi(y) < 1/y$. This inequality is confirmed in [Gordon \(1941\)](#). Furthermore, note that $\Phi(0) / \varphi(0) = \sqrt{\pi/2}$.

The combination of Claim 27 and Claim 28 implies that $\frac{\partial \Pi}{\partial P} > 0$ when $P < V + y - \frac{\kappa\delta}{2}$ if

$$\frac{(W - P) \sqrt{\tau_\theta}}{1 - \kappa\delta \Delta \varphi(\Delta[\theta^*(P) - y])} - \sqrt{\frac{\pi}{2}} > 0 \quad (2.21)$$

Therefore, a sufficient condition for (2.21) is $P < W - \sqrt{\pi/2\tau_\theta}$. By setting $\frac{\partial \Pi}{\partial P} = 0$ we obtain the expression in (2.13), where it is possible to see that $P^* < W - \sqrt{\pi/2\tau_\theta}$.

Now the concavity of Π for $P \geq V + y - \frac{\kappa\delta}{2}$ is sufficient to guarantee a unique interior solution. Notice that

$$\frac{\partial^2 \Pi}{\partial P^2} = -2 \frac{\partial \beta(\theta^*(P))}{\partial P} + (W - P) \frac{\partial^2 \beta(\theta^*(P))}{\partial P^2} \quad (2.22)$$

where $\frac{\partial \beta(\theta^*(P))}{\partial P} > 0$ and

$$\frac{\partial^2 \beta(\theta^*(P))}{\partial P^2} = \left[\sqrt{\tau_\theta} \frac{\partial \theta^*}{\partial P} \right]^2 \varphi'(\sqrt{\tau_\theta}[\theta^*(P) - y]) + \varphi(\sqrt{\tau_\theta}[\theta^*(P) - y]) \sqrt{\tau_\theta} \left(\frac{\partial^2 \theta^*}{\partial P^2} \right) \quad (2.23)$$

Given $P \geq V + y - \frac{\kappa\delta}{2}$, it must be that $\theta^*(P) - y > 0$ by Claim 27 and therefore $\varphi'(\sqrt{\tau_\theta}[\theta^*(P) - y]) < 0$. Moreover, algebra confirms that $\frac{\partial^2 \theta^*}{\partial P^2} \leq 0$.

Finally, notice that we require $V + y - \frac{\kappa\delta}{2} < W - \sqrt{\pi/2\tau_\theta}$ and that $P^* \in \left[V + y - \frac{\kappa\delta}{2}, W - \sqrt{\pi/2\tau_\theta} \right]$. ■

2.7.8 Proof of Proposition 24

A) **Proof** $\frac{dP}{dy} \geq 0$

The optimal bid P^* satisfies

$$\begin{aligned}\beta(\theta^*(P)) &= (W - P) \left[\frac{\partial \beta(\theta^*(P))}{\partial \theta^*} \frac{\partial \theta^*(P)}{\partial P} \right] \\ &= (W - P) \left(\frac{\sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta} [\theta^*(P) - y])}{1 - \kappa \delta \Delta \varphi(\Delta [\theta^*(P) - y])} \right)\end{aligned}\quad (2.24)$$

Differentiate both sides with respect to y so as to get $\frac{dLHS}{dy} = \frac{dRHS}{dy}$. Note that $\beta(\theta^*(P(y), y), y)$. Hence

$$\frac{dLHS}{dy} = \frac{\partial \beta}{\partial \theta^*} \left(\frac{\partial \theta^*}{\partial P} \frac{dP}{dy} + \frac{\partial \theta^*}{\partial y} \right) + \frac{\partial \beta}{\partial y} \quad (2.25)$$

and

$$\begin{aligned}\frac{dRHS}{dy} &= -\frac{dP}{dy} \left(\frac{\sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta} (\theta^* - y))}{1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))} \right) + \frac{(W - P) \sqrt{\tau_\theta}}{[1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))]^2} \\ &\quad \cdot \left\{ \begin{aligned} &\left(\frac{\partial \theta^*}{\partial P} \frac{dP}{dy} + \frac{\partial \theta^*}{\partial y} - 1 \right) \sqrt{\tau_\theta} \varphi'(\sqrt{\tau_\theta} (\theta^* - y)) [1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))] \\ &+ \varphi(\sqrt{\tau_\theta} (\theta^* - y)) \left(\frac{\partial \theta^*}{\partial P} \frac{dP}{dy} + \frac{\partial \theta^*}{\partial y} - 1 \right) \kappa \delta \Delta^2 \varphi'(\Delta (\theta^* - y)) \end{aligned} \right\}\end{aligned}$$

Rewriting we have

$$\begin{aligned}\frac{dRHS}{dy} &= -\frac{dP}{dy} \frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial P} + \left(\frac{\partial \theta^*}{\partial P} \frac{dP}{dy} + \frac{\partial \theta^*}{\partial y} - 1 \right) \cdot \\ &\quad \underbrace{\frac{(W - P) \sqrt{\tau_\theta}}{[1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))]^2} \left\{ \begin{aligned} &\sqrt{\tau_\theta} \varphi'(\sqrt{\tau_\theta} (\theta^* - y)) [1 - \kappa \delta \Delta \varphi(\Delta (\theta^* - y))] \\ &+ \varphi(\sqrt{\tau_\theta} (\theta^* - y)) \kappa \delta \Delta^2 \varphi'(\Delta (\theta^* - y)) \end{aligned} \right\}}_{\Psi}\end{aligned}$$

where $\Psi < 0 \iff \theta^* > y$. Rearranging we get

$$\begin{aligned}\frac{dRHS}{dy} &= -\frac{dP}{dy} \frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial P} + \left(\frac{\partial \theta^*}{\partial P} \frac{dP}{dy} + \frac{\partial \theta^*}{\partial y} - 1 \right) \Psi \\ &= \frac{dP}{dy} \left(\frac{\partial \theta^*}{\partial P} \Psi - \frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial P} \right) + \Psi \left(\frac{\partial \theta^*}{\partial y} - 1 \right)\end{aligned}\quad (2.26)$$

Finally, $\frac{dLHS}{dy} = \frac{dRHS}{dy}$ yields

$$\begin{aligned} \frac{dP}{dy} \frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial P} + \frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial y} + \frac{\partial \beta}{\partial y} &= \frac{dP}{dy} \left(\frac{\partial \theta^*}{\partial P} \Psi - \frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial P} \right) + \Psi \left(\frac{\partial \theta^*}{\partial y} - 1 \right) \\ \frac{dP}{dy} \underbrace{\left[2 \underbrace{\frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial P}}_{+} - \underbrace{\frac{\partial \theta^*}{\partial P} \Psi}_{+} \right]}_{+} &= \underbrace{\underbrace{\Psi}_{-} \left(\underbrace{\frac{\partial \theta^*}{\partial y} - 1}_{-} \right) - \underbrace{\frac{\partial \beta}{\partial \theta^*} \frac{\partial \theta^*}{\partial y}}_{+} - \underbrace{\frac{\partial \beta}{\partial y}}_{-}}_{+} \end{aligned} \quad (2.27)$$

B) **Proof** $\frac{d\beta}{dy} \leq 0$

Note that $\beta(\theta^*(P(y), y), y)$. Hence, differentiating both sides of the expression above we have

$$\frac{d\beta}{dy} = \frac{\partial \beta}{\partial \theta^*} \left(\frac{\partial \theta^*}{\partial P} \frac{\partial P}{\partial y} + \frac{\partial \theta^*}{\partial y} \right) + \frac{\partial \beta}{\partial y}$$

We can plug in all corresponding expressions in the *RHS* with the exception of $\frac{\partial P}{\partial y}$.

Then,

$$\begin{aligned} \frac{d\beta}{dy} &= \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y)) \left[\frac{\left(\frac{\partial P}{\partial y} \right) - \kappa \delta \Delta \varphi(\Delta[\theta^* - y])}{1 - \kappa \delta \Delta \varphi(\Delta[\theta^* - y])} \right] - \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y)) \\ &= \frac{\sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y))}{1 - \kappa \delta \Delta \varphi(\Delta[\theta^* - y])} \left[\left(\frac{\partial P}{\partial y} \right) - 1 \right] \end{aligned}$$

$$\text{So } \frac{d\beta}{dy}_{sign} = \left(\frac{\partial P}{\partial y} \right) - 1.$$

Now recall

$$P = W - \frac{[1 - \kappa \delta \Delta \varphi(\Delta(\theta^* - y))] \Phi(\sqrt{\tau_\theta}(\theta^* - y))}{\sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y))}$$

and let

$$F(P, y) = P - W + \frac{[1 - \kappa \delta \Delta \varphi(\Delta(\theta^* - y))] \Phi(\sqrt{\tau_\theta}(\theta^* - y))}{\sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y))}$$

so we can use the IFT $\frac{\partial P}{\partial y} = -\frac{\partial F(P, y)/\partial y}{\partial F(P, y)/\partial P}$. For this we fix P , so we consider P rather

than $P(y)$ and $\theta^*(y)$ rather than $\theta^*(P(y), y)$.

First, compute $\frac{\partial F(P, y)}{\partial y}$:

$$\frac{\partial F(P, y)}{\partial y} = \frac{(A + B) \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y)) + C}{\tau_\theta [\varphi(\sqrt{\tau_\theta}(\theta^* - y))]^2} < 0$$

where

$$A \equiv - \left(\frac{\partial \theta^*}{\partial y} - 1 \right) \kappa \delta \Delta^2 \varphi'(\Delta(\theta^* - y)) \Phi(\sqrt{\tau_\theta}(\theta^* - y)) < 0$$

$$B \equiv [1 - \kappa \delta \Delta \varphi(\Delta(\theta^* - y))] \left(\frac{\partial \theta^*}{\partial y} - 1 \right) \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y)) < 0$$

$$C \equiv -[1 - \kappa \delta \Delta \varphi(\Delta(\theta^* - y))] \Phi(\sqrt{\tau_\theta}(\theta^* - y)) \left(\frac{\partial \theta^*}{\partial y} - 1 \right) \tau_\theta \varphi'(\sqrt{\tau_\theta}(\theta^* - y)) < 0$$

Second, compute $\frac{\partial F(P, y)}{\partial P}$:

$$\frac{\partial F(P, y)}{\partial P} = 1 + \frac{-(A + B) \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y)) - C}{\tau_\theta [\varphi(\sqrt{\tau_\theta}(\theta^* - y))]^2} > 0$$

using the fact that $\frac{\partial \theta^*}{\partial y} - 1 = -\frac{\partial \theta^*}{\partial P}$.

Now

$$\frac{\partial P}{\partial y} = - \frac{(A + B) \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y)) + C}{\tau_\theta [\varphi(\sqrt{\tau_\theta}(\theta^* - y))]^2 - (A + B) \sqrt{\tau_\theta} \varphi(\sqrt{\tau_\theta}(\theta^* - y)) - C} \in [0, 1)$$

■

2.8 Appendix B: Robustness

2.8.1 Cross-Ownership

Suppose that a proportion λ of sophisticated shareholders own stock of the Bidder. These shareholders obtain an additional payoff $B \in \mathbb{R}$ if the takeover succeeds. Denote *common* shareholders those that own stock only in the target company and *bidding* shareholders those who also own stock of the Bidder. Then, the volume of stock sales reads $\rho = \delta[\lambda\gamma_B + (1 - \lambda)\gamma_C]$.

We derive the equilibrium as in [Section 2.3](#). However, we initially assume that the two groups of sophisticated shareholders (common and bidding) have different decision thresholds: x_C^* and x_B^* . An analysis analogous to the Proof of [Lemma 15](#) leads us to conclude that if common shareholders have an x_C^* -threshold strategy and bidding shareholders have an x_B^* -threshold strategy, then the Board has θ^* -threshold behavior for $\kappa < \frac{\sqrt{2\pi}}{\sqrt{\tau_\varepsilon}\delta}$. This satisfies

$$\theta^* = P - V + \kappa\delta \left[(1 - \lambda) \underbrace{[1 - \Phi(\sqrt{\tau_\varepsilon}[x_C^* - \theta^*])]}_{\rho_C} + \lambda \underbrace{[1 - \Phi(\sqrt{\tau_\varepsilon}[x_B^* - \theta^*])]}_{\rho_B} \right] \quad (2.28)$$

The probability of a takeover and the price set by market makers are hence defined as in [\(2.6\)](#) and [\(2.7\)](#) respectively.

Now consider the benefit to selling over holding for a bidding investor receiving a signal x_{i_B} when other shareholders follow a threshold strategy:

$$\begin{aligned} u(x_{i_B}, x_B^*, x_C^*) &= \left\{ \begin{array}{l} (M + B) \Pr[\theta \leq \theta^* | x_{i_B}] + M \Pr[\theta > \theta^* | x_{i_B}] \\ - (P + B) \Pr[\theta \leq \theta^* | x_{i_B}] - V \Pr[\theta > \theta^* | x_{i_B}] \end{array} \right\} \\ &= M - (P - V) \Phi \left(\sqrt{\tau_\theta + \tau_\varepsilon} \left(\theta^* - \frac{\tau_\theta y + \tau_\varepsilon x_{i_B}}{\tau_\theta + \tau_\varepsilon} \right) \right) - V \end{aligned} \quad (2.29)$$

Using the argument of Proof of [Lemma 16](#), for any given x_C^* there is only one solution $u(x_B^*, x_B^*, x_C^*) = 0$ if $\kappa < \frac{\sqrt{2\pi}}{\sqrt{\tau_\varepsilon}\delta}$.

Note that the payoff B is irrelevant to the bidding shareholder's decision. As a consequence, the two types of shareholders face the same problem and therefore they share the same solution:

$$u(x_{i_C}, x_B^*, x_C^*) = u(x_{i_B}, x_B^*, x_C^*) \Rightarrow u(x_C^*, x_B^*, x_C^*) = u(x_B^*, x_B^*, x_C^*) \Rightarrow x_B^* = x_C^* \quad (2.30)$$

Finally, note that the proportion of sales is equal for common shareholders and bidding shareholders, i.e. $\gamma_C = \gamma_B$. Thus the proportion of each type of shareholder, captured by λ , becomes irrelevant to the equilibrium result, and Proposition 17 follows.

2.8.2 Informed External Investors

The common prior about the Board's type is $\theta \sim N(y, 1/\tau_\theta)$. While sophisticated shareholders do not have additional information, there is a continuum of external investors, e.g. risk-arbitrageurs and hedge fund activists, of mass one where each investor j receives an *iid* signal $x_j = \theta + \varepsilon_j$ with $\varepsilon_j \sim N(0, 1/\tau_\varepsilon)$ and updates beliefs accordingly. The interim stock price M is set by market makers with zero expected profits. External investors acquire stock at the interim if their value is higher than M , i.e. if they receive a sufficiently low signal of the Board's type.

The equilibrium analysis is analogous to that in Section 2.3. For clarity, we denote our endogenous variables in this section with the subindex I .

Suppose that investors follow an x_I^* -threshold strategy: buy in the offer period if and only if they observe $x < x_I^*$. Then stock sales equal the mass of investors that received a signal below the threshold relative to the number of sophisticated shareholders, $\rho_I = \delta \Phi(\sqrt{\tau_\varepsilon}(x^* - \theta))$. As a result, the Board has θ_I^* -threshold equilibrium behavior. We do not require a uniqueness condition, in contrast to our benchmark setting, as θ_I^* is unambiguously increasing in x_I^* .

Market makers have zero expected profits and only observe public informa-

tion. Thus, both the probability of a takeover β_I and the interim stock price M_I are characterised by expressions equivalent to (2.6) and (2.7) respectively.

Consider now the strategy of an investor j observing signal x_j and hence with an updated belief $\theta|x_j \sim N\left(\frac{\tau_\theta y + \tau_\varepsilon x_j}{\tau_\theta + \tau_\varepsilon}, \frac{1}{\tau_\theta + \tau_\varepsilon}\right)$. If other investors follow an x_I^* -threshold strategy, her expected benefit to buying over not buying is

$$\begin{aligned} u(x_j, x_I^*) &= (P - V) \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta_I^* - \frac{\tau_\theta y + \tau_\varepsilon x_j}{\tau_\theta + \tau_\varepsilon}\right)\right) + V - M \\ &= (P - V) \left[\Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta_I^* - \frac{\tau_\theta y + \tau_\varepsilon x_j}{\tau_\theta + \tau_\varepsilon}\right)\right) - \Phi(\sqrt{\tau_\theta}(\theta_I^* - y)) \right] \end{aligned} \quad (2.31)$$

It is possible to see that the investor's utility of buying is monotonically decreasing in the signal received x_j . Thus, the game of external investors is opposite to the game of sophisticated shareholders (characterised in the benchmark model), whose utility of selling is increasing as a function of the signal they receive. With an argument analogous to the one developed in Proof of Lemma 16, we find that there exists a unique solution to $u(x_j, x_I^*) = 0$ and hence external investors have a unique x_I^* -threshold equilibrium. The equilibrium tuple $\{\theta_I^*, x_I^*\}$ is characterised in the next proposition:

Proposition 29 *There exists a unique Bayesian Nash Equilibrium in which all external investors buy shares if and only if they observe a signal below x_I^* . The takeover succeeds if, and only if, the takeover resistance is below the threshold θ_I^* . The thresholds are characterised implicitly by the following equations:*

$$\theta_I^* = P - V + \kappa\delta [1 - \Phi(\Delta(\theta_I^* - y))] \quad (2.32)$$

$$x_I^* = \theta_I^* - \frac{\Delta}{\sqrt{\tau_\varepsilon}}(\theta_I^* - y) \quad (2.33)$$

Proof. Analogous to Proof of Proposition 17. ■

As a result, stock sales at the interim are $\rho_I = \delta\Phi(\sqrt{\tau_\varepsilon}(\theta_I^* - \theta) - \Delta(\theta_I^* - y))$. Comparing θ_I^* with the threshold in (2.9) it is possible to appreciate that revers-

ing the information structure switches the mass of strategic decisions influencing the takeover, i.e. $1 - \Phi(\Delta(\theta_I^* - y))$. In particular, the proportion of sophisticated shareholders selling now corresponds to the probability that an investor receives a signal $x < x_I^*$. This is in contrast to the benchmark model, where the proportion of sophisticated shareholders selling equals the probability that $x > x^*$. As a consequence, while the effect of the fundamental variables on both the probability of a takeover and the interim stock price has the same sign as in the original setting, their effect on the volume of sales is different. The following can be shown emulating the method of proof used in the prior analysis:

Proposition 30 *Responses of the main outcomes to marginal increases of the model fundamentals are as follows*

<i>Fundamental</i>	<i>Effect on outcomes</i>		
	<i>Pr. Takeover β</i>	<i>Interim Price M</i>	<i>Stock Sales ρ</i>
<i>Bid premium: $P - V$</i>	+	+	+
<i>New-shareholder pressure: κ</i>	+	+	+
<i>Expected Board resistance: y</i>	−	−	+

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Chapter 3

Competing under financial constraints

3.1 Introduction

Firms' limited access to credit is a widespread phenomenon. Contractual frictions may generate incentives for borrowers to strategically default on their debt, and lenders often respond by restraining credit. A firm is financially constrained when it cannot secure as much credit as it wishes, even though it is willing to pay it back ([Tirole 2010](#)). It is also well known that borrowers' competitive pressure can exacerbate financial constraints. In particular, product market competition typically reduces the profitability of investment, thereby increasing the agency costs of a financial transaction and further limiting access to credit. The common wisdom is therefore that financial constraints harm firms' ability to compete in the product market (e.g., [Holmstrom and Tirole 1997](#)). In this paper I challenge such view.

The idea here is that credit constraints both affect and are affected by commercial relationships, and the pricing policy of a strategic supplier may depend on the financing conditions of its customer (retailer). More specifically, input prices determine the optimal investment level of a retailer and, in turn, whether the re-

tailer is financially constrained. Furthermore, the retailer's investment capacity has an impact on market outcomes and profits, thereby affecting the surplus that can be extracted by its supplier and thus, the pricing policy. I characterize the interplay between input prices and retailers' financial constraints. I show that a financially constrained retailer faces lower prices, and this can put it at a competitive advantage, increasing profits.

The effect of firms' debt on their ability to compete has been largely studied, and different conclusions are reached.¹ Notably, [Brander and Lewis \(1986\)](#) and [Maksimovic \(1988\)](#) show that debt provides an incentive to compete more aggressively in future periods, thereby acting as a commitment device that can increase profits. In contrast, [Bolton and Scharfstein \(1990\)](#) argue that a company's leverage may trigger predatory strategies by competitors, inducing the company to default on its debt and exit the market. Despite divergent arguments, these studies consistently consider leverage to be a signal of the firm's type as a competitor in the product market, and therefore it affects equilibrium market outcomes.

In this context, it is natural to consider that firms' ability to compete affects their debt capacity, therefore creating an endogenous relation between leverage and product market competition. I study a trade credit transaction between a supplier and a (potentially) financially constrained retailer to shed light on this relation. Trade credit occurs when a supplier allows a customer (retailer) to pay with delay for goods already delivered. This provides a convenient framework for my analysis because the lender is also a supplier, and the financial transaction may affect the commercial relationship. Moreover, evidence shows that trade credit is an important source of firms' finance, also in countries with well-developed financial systems (e.g. [Giannetti 2003](#); [Rajan and Zingales 1995](#)).

I model the relation between a retailer that competes in the product market and a supplier extending trade credit. I follow [Burkart and Ellingsen \(2004\)](#) in

¹See [Cestone \(1999\)](#) for a review of the literature on corporate financing and product market competition.

assuming that the retailer's investment in the product market is not contractible, and limited liability creates an incentive to divert the input without honouring the debt. As a consequence, the supplier must limit the line of credit to make repayment incentive compatible, and this may generate financial constraints for the retailer. Crucially, here the input price is strategically set by the supplier, and credit is the product of price and quantity of input borrowed. Thus, for a given level of financial constraints, a higher (lower) input price reduces (increases) the quantity of input that can be extended in credit. Moreover, both the price and the quantity of input borrowed affect the relative profitability of diverting, and hence determine whether the retailer is financially constrained. As a result, financial constraints arise endogenously in equilibrium.

In my model leverage has no effect on market outcomes in the absence of contractual frictions. With no financial constraints, the vertical relation between the supplier and the retailer is characterized by double marginalization, and market outcomes in the retail market are those of a Cournot-Nash equilibrium. In contrast, in the equilibrium with financial constraints the retailer exhausts the line of credit and double marginalization vanishes. This triggers two key effects for my results. First, the supplier can extract all retailer's surplus conditional on debt repayment being incentive compatible. As a consequence, the profits of the retailer equal its agency rents. Second, financial constraints act as a commitment device for the retailer's production, and the supplier sets an input price that yields the production of a Stackelberg leader. For intermediate levels of competitive pressure, retailer's agency rents exceed the profits that it would make in a setting with no contractual frictions —double marginalization. Then, financial constraints are profitable for the retailer.

Whether financial constraints arise in equilibrium depends on market fundamentals. I focus on the intensity of the retailer's competitive pressure. With low competition, production is relatively profitable and agency costs are small. Then,

debt repayment is incentive compatible in a setting with double marginalization and the retailer is not financially constrained. Conversely, high competition increases incentives to divert and, under double marginalization, repayment is not incentive compatible, leading to financial constraints. Several papers have studied the effects of supplier's (lender's) product market competition on the extension of trade credit (e.g., [Petersen and Rajan 1997](#); [McMillan and Woodruff 1999](#); [Fisman and Raturi 2004](#); [Fabbri and Klapper 2016](#)). However, little attention has been paid to the effects of the retailer's competitive pressure. Unsurprisingly, in my model retailer's competition reduces the value of credit offered by the supplier because it lowers the profitability of producing and increases agency costs. This, nonetheless, need not imply that less input can be borrowed, as the reduction might be driven by a lower input price.

When contractual frictions lead to financial constraints, they affect market outcomes through both the retailer's marginal cost (input price) and its production capacity (quantity of input extended in credit). The literature has studied the role of these two on a firm's own, and rivals', production choices. A representative work is [Dixit \(1980\)](#), which shows that a firm can invest in capacity to lower the relevant marginal cost and increase production, potentially deterring the entrance of competitors.² Here, the operating mechanism is totally different because both marginal cost and capacity are determined by a strategic supplier, which needs to offer a line of credit so that repayment is incentive compatible. This setting is close to [Fershtman and Judd \(1987\)](#), who study the incentive contracts that owners (principals) choose for their managers (agents) in an oligopolistic context. I analyse the product market effects of an agency problem where a supplier effectively delegates production to a retailer because it cannot access the market itself.

The strategic role of input prices drives the main results of the paper. Input

²The work of [Dixit \(1980\)](#) is followed by a number of papers studying the effects of capacity on the product market. These include [Kirman and Masson \(1986\)](#), [Kulatilaka and Perotti \(1998\)](#), [Reynolds \(1991\)](#). Notably, [Leach et al. \(2013\)](#) study the interaction of debt and capacity commitments.

price determines both the retailer's optimal production and the profitability of participating in the retail market. Thus, it also regulates the incentives to divert, and therefore the quantity of input that the supplier can extend in credit while making repayment incentive compatible. Other work in trade credit has studied the strategic role of prices in a different context. For example, [Smith \(1987\)](#) and [Brennan et al. \(1988\)](#) show that price discrimination can be used to reveal the creditworthiness of different retailers. [Daripa and Nilsen \(2011\)](#) demonstrate that inter-firm credit may subsidise inventory holding costs, and evaluate input price adjustment as an alternative to the extension of credit. However, as noted in [Giannetti et al. \(2011\)](#), it is still not clear whether a supplier would reduce input prices to a credit-constrained retailer. I answer this question.

To characterize the endogenous relation between leverage and product market competition, I assume that input prices are linear. This yields two different types of equilibria (constrained and unconstrained) that depend on market fundamentals. I conduct comparative statics on the levels of competitive pressure to study how these determine equilibrium outcomes. Then I study the same model when the supplier can set non-linear prices. I show that market outcomes are equal to those of the equilibrium with financial constraints in the main setting (linear prices), but the retailer need not be constrained. More specifically, the supplier can always extract all the retailer's surplus conditional on repayment being incentive compatible, and the line of credit acts as a commitment device because the retailer exhausts it. However, the retailer may never be constrained because optimal production is determined by the marginal price of input, and with a non-linear tariff this is not relevant for the supplier's profit maximization.

The remainder of the paper is organized as follows. Section 2 describes the model and Section 3 characterizes the equilibrium. Section 4 conducts comparative statics on the retailer's competitive pressure and interprets the results. Section 5 considers a setting with non-linear prices. Section 6 concludes.

3.2 The Model

I consider the vertical relation between a penniless *retailer* and an input *supplier* who can extend trade credit, and how this relationship is shaped by competition in the retailer's product market. The retailer needs to borrow input in order to enter the product market, which is operated by N other firms that I call *incumbents*. Contractual frictions in the vertical relation allow the retailer to divert the input borrowed without honouring his debt, and this limits the volume of credit that the supplier can extend while keeping repayment incentive compatible. There are three dates $t = 0, 1, 2$. There is no discounting and all agents are risk neutral.

At $t = 0$ the supplier sets a price ω per each unit of input. The price is observable to all market participants, who also know that the supplier must incur a cost c_s to produce each unit of input. In Section 5 I relax the assumption that prices are linear and discuss its implications.

At $t = 1$ the supplier extends trade credit of up to L units of input to the retailer. This can be paid for at the end of the game. For simplicity, I assume that there is no interest on the loan. Moreover, I ease presentation by assuming that (i) the retailer cannot borrow money from a financial institution, hence the supplier is the only potential source of credit; (ii) the retailer has zero funds, being forced to borrow all the input that he wants to invest. Neither of these assumptions affect the results qualitatively.³ After observing the offer L , the retailer borrows $I \leq L$ units of input, incurring a debt ωI . Neither the offer L nor the transaction I are observable to other parties. The retailer can transform each unit of input into a unit of output costlessly.

At $t = 2$ retailer and incumbents simultaneously decide their production.

³(i) In line with [Burkart and Ellingsen \(2004\)](#), money is more divertible than input, so agency problems are stronger in standard credit transactions than in trade credit transactions. As a result, suppliers can extend credit in situations where financial institutions are not able to do so. Introducing standard credit would diminish, but not eliminate, the role of the supplier as a lender. (ii) With a positive amount of funds, the retailer would have to borrow a smaller quantity of input, if any, and thus financial constraints would arise for a smaller range of parameters.

The retail market is characterized by an inverse demand function $P(Q) = M - Q$, where Q captures the total quantity of homogeneous product that is sold in it. In particular, $Q = \sum_{i=1}^N q_i + q_e$, where q_i represents the quantity produced by incumbent $i \in \{0, 1, 2, \dots, N\}$ and $q_e \leq I$ the quantity produced by the retailer. All incumbents have the same production cost c per unit of output.

Crucially, the retailer may divert the input borrowed rather than investing it in the retail market and honouring the debt. I assume that while both output and sales revenues are verifiable and can therefore be pledged to the supplier, neither the input purchase nor the investment decision are contractible. The retailer has limited liability, so the debt is honoured only to the extent of market revenues. These, in turn, can only be enjoyed by the retailer after honouring repayment obligations. Each unit of input diverted generates a private benefit $\beta < \min\{c, c_s\}$, where the relatively low revenue reflects the inefficiency of diverting. Furthermore, for the agency problem to affect equilibrium outcomes I assume that the supplier is not “too efficient,” so it holds that $\beta + c_u > c$.

From the previous assumptions it follows that the net profits of the retailer read

$$\pi_e = \max \left\{ \left(M - \sum_{i=1}^N q_i - q_e \right) q_e - \omega I, 0 \right\} + \beta(I - q_e). \quad (3.1)$$

Here, the first term within the maximum operator captures net market profits, and zero is the lower bound guaranteed by limited liability. The last term in (3.1) represents the revenues from diversion: β for each unit of input borrowed and not invested in the retail market, i.e., for $I - q_e$ units. Note that if the supplier does not limit the credit line to L , the retailer’s best response is to borrow an unlimited amount of input and divert it.

Formally, I consider the following definition of financial constraints:

Definition 31 *Let $q_e^u(\omega)$ denote the optimal retailer’s production in the absence of incentives to divert for a given input price ω . The retailer is financially constrained*

when it cannot borrow enough input to produce optimally, i.e., $L < q_e^u(\omega)$.

The definition captures the essence of credit rationing, which [Bester and Hellwig \(1987\)](#) described as “a would-be borrower is said to be rationed if he cannot obtain the loan that he wants even though he is willing to pay the interest that the lenders are asking, perhaps even a higher interest.”⁴ Notably, Definition 31 shows that whether the retailer is financially constrained not only depends on the line of credit L , but also on the optimal investment, and hence on the input price ω .

Parameter β captures input liquidity. This is, the profitability of allocating input to alternative uses for private benefit. In the presence of contractual frictions, large β increases agency costs and may lead to tighter financial constraints. Evidence by [Cunat \(2007\)](#) and [Giannetti et al. \(2011\)](#) suggests that input liquidity is related to product characteristics. In particular, generic inputs are easy to divert and thus highly liquid, whereas more tailored inputs have low value in alternative markets, thereby alleviating agency costs.

The agency problem here is akin to [Burkart and Ellingsen \(2004\)](#), who provide a rationale for trade credit. I extend their setting in two key dimensions that allow me to study the interaction of financial constraints and product market competition. First, I endogenize input price. By setting ω , the supplier determines both the retailer’s optimal production in the retail market and his incentives to divert the input and not honouring his debt. As a result, both the demand for input and the line of credit L are a function of ω , and financial constraints arise endogenously in equilibrium. Second, I introduce product market competition in the retail market. Parameter N captures competitive pressure, which crucially affects the profitability of investment and in turn, the input price. Assuming that incumbents are identical simplifies the analysis while preserving the key role of competition.

I assume that the supplier is a monopolist to the retailer, which in turn represents the only access to the retail market. This is consistent with [Petersen](#)

⁴This quote opens the discussion of the chapter ‘Outside Financing Capacity’ in [Tirole \(2010\)](#)

and Rajan (1997), McMillan and Woodruff (1999) and Cunat (2007), who find that more trade credit is extended where suppliers have low competition. The scarcity of alternative sources of input increases the value of the commercial relationship for retailers and lowers the incentives to strategically default on their debt. This reduces agency costs and enables the extension of trade credit by suppliers. Formally, the supplier's market power allows me to study how agency problems affect input prices and thus the competitiveness of the retailer in the product market.

3.3 Equilibrium Analysis

I consider subgame perfect Nash equilibria of this model. I start with a preliminary result that facilitates subsequent analysis, and then solve recursively:

Lemma 32 *In equilibrium, the retailer pays all his debt and does not allocate any input to alternative uses, i.e., $q_e = I$.*

The result is intuitive. The retailer's profit function (3.1) shows that honouring the debt is an *all-or-nothing* decision. This property is shared with Burkart and Ellingsen (2004), and follows from the assumption that market revenues are contractible. More specifically, when market revenues are smaller than debt value ωI , the profitability of investment is zero. Alternatively, the same input might be diverted, generating a net profit of $\beta > 0$ per unit. Thus, either the retailer produces enough output to enjoy market revenues after repayment, or diverts it all. In equilibrium, the supplier only extends credit when repayment is incentive compatible. Therefore, whenever credit is extended, the debt is fully honoured.

Lemma 32 establishes that, besides honouring the debt in full, the retailer does not borrow any input to allocate it elsewhere. In particular, one may consider a case where the retailer finds it profitable to use a fraction of input borrowed for production —and pay the debt, and to allocate the remaining part to an alternative use, generating a unit revenue of β . Notice that this is never the case because in

equilibrium the input price must be such that $\omega \geq c_u$, whereas we have $c_u > \beta$ by assumption. In words, the supplier always sets an input price weakly higher than its cost, which in turn is higher than the revenues of allocating the input to alternative uses. Thus, given a contract such that repayment is incentive compatible, it cannot be profitable for the retailer to buy input and allocate it elsewhere.

3.3.1 Production

At $t = 2$ retailer and incumbents simultaneously make production decisions. Their unit costs — ω and c respectively, are known to all market participants. Moreover, the retailer's production is limited to $I \leq L$, where neither the credit line offered by the supplier nor the quantity of input borrowed have been observed by the incumbents.

Incumbent i maximizes net profits $\pi_i = (M - \sum_{-i} q_{-i} - q_i - q_e) q_i - c q_i$. Here, $\sum_{-i} q_{-i}$ represents for the production of all other incumbents. By symmetry, each of them produces the same quantity, which satisfies the best reply

$$q_i(q_e) = \frac{M - q_e - c}{N + 1}, \forall i. \quad (3.2)$$

Thus, the sum of all incumbents' production can be represented as Nq_i .

From Lemma 32 it follows that retailer's profits are those obtained by producing with a unit cost ω . More specifically, input price and line of credit must be such that all input borrowed is used for production, i.e., $q_e = I$. As a result, retailer's profit function (3.1) collapses to $\pi_e = (M - Nq_i - q_e) q_e - \omega q_e$. Moreover, production levels are upper bounded by the line of credit L . I denote the retailer's unconstrained best reply by $q_e^u(q_i)$. Then, the best reply reads

$$q_e(q_i) = \begin{cases} q_e^u(q_i) = \frac{M - Nq_i - \omega}{2} & \text{when } q_e^u(q_i) \leq L \\ L & \text{when } q_e^u(q_i) > L \end{cases}. \quad (3.3)$$

Response function (3.3) shows that the retailer's financial constraint effectively acts as a capacity constraint. Given an input price ω , the retailer's optimal production is $q_e^u(q_i)$, and he is financially constrained when $q_e^u(q_i) > L$. The concavity of market profits implies that whenever the retailer is financially constrained, he exhausts all credit, i.e., $q_e(q_i) = L$.

Incumbents do not observe the line of credit offered by the supplier at $t = 1$, but they have a conjecture that is correct in equilibrium. As will become clear, there exists a one-to-one mapping from the publicly observed input price set by the supplier at $t = 0$ to the line of credit offered at $t = 1$. I derive the equilibrium production policies of market participants for a given L , and then show that the incumbents' conjecture is correct. For simplicity, my notation does not differentiate between the conjecture of L and the actual line of credit.

Lemma 33 *Suppose that incumbents' conjecture of the credit line L is correct in equilibrium. Then, equilibrium production levels of the retailer and the incumbents satisfy*

$$q_e^* = \begin{cases} q_e^u = \frac{M-(N+1)\omega+Nc}{N+2} & \text{when } q_e^u \leq L \\ L & \text{when } q_e^u > L \end{cases}, \quad (3.4)$$

$$q_i^* = \begin{cases} q_i^u = \frac{M-2c+\omega}{N+2} & \text{when } q_e^u \leq L \\ q_i^c = \frac{M-L-c}{N+2} & \text{when } q_e^u > L \end{cases} \quad (3.5)$$

and the corresponding profits are

$$\pi_e^* = \begin{cases} \pi_e^u = \left[\frac{M-(N+1)\omega+Nc}{N+2} \right]^2 & \text{when } q_e^u \leq L \\ \pi_e^c = L \left[\frac{M-(N+1)\omega-L+Nc}{N+1} \right] & \text{when } q_e^u > L \end{cases}, \quad (3.6)$$

$$\pi_i^* = \begin{cases} \pi_i^u = \left[\frac{M-2c+\omega}{N+2} \right]^2 & \text{when } q_e^u \leq L \\ \pi_i^c = \left[\frac{M-c-L}{N+1} \right]^2 & \text{when } q_e^u > L \end{cases} \quad (3.7)$$

Optimal production $\{q_e^*, q_i^*\}$ is obtained by solving for best replies (3.2) and

(3.3); the profits functions follow from plugging the corresponding solutions into π_e and π_i . I derive the expressions with more detail in the Appendix.

For a given input price ω , production in the unconstrained equilibrium $\{q_e^u, q_i^u\}$ is that of a standard simultaneous Cournot-Nash game. Here, q_e^u corresponds to the optimal production used in Definition 31. For exposition, I no longer specify that it is a function of ω . Notably, market outcomes in the constrained equilibrium $\{L, q_i^c\}$ are those of a game where the financially constrained retailer becomes a leader that *must* choose production L . Thus, the constraint can potentially provide a first-mover advantage. However, as will become clear, L is itself a function of ω , strategically set by the supplier.

Profits functions (3.6) show that, given a price ω and a line of credit L , financial constraints can only harm the retailer. Formally, simple algebra reveals that $\pi_e^c > \pi_e^u \iff L > q_e^u$. In words, the retailer's profits in the constrained equilibrium can only exceed those in the unconstrained one when the retailer is not constrained, i.e., never. This is not surprising because, everything else equal, financial constraints can only limit the retailer's ability to maximize profits. Nonetheless, in equilibrium both input price and credit line are set by the supplier and depend, in turn, on whether the retailer is financially constrained. I study these in the next sections.

3.3.2 Line of credit

At $t = 1$ the supplier offers a credit line L to the retailer, and the retailer borrows $q_e \leq L$. The following Lemma characterizes the equilibrium line of credit as a function of input price and market fundamentals.

Lemma 34 *Given input price ω , in equilibrium the credit line L makes the retailer indifferent between producing and diverting all input without honouring the debt,*

i.e., it satisfies

$$L^* = \begin{cases} L^u = \frac{1}{\beta} \left[\frac{M-(N+1)\omega+Nc}{N+2} \right]^2 & \text{when } \pi_e^u \geq \beta q_e^u \\ L^c = M - (N+1)(\omega + \beta) + Nc & \text{when } \pi_e^u < \beta q_e^u \end{cases}. \quad (3.8)$$

Moreover, incumbents' conjecture of L is correct.

In the Appendix I derive (3.8); here I explain why this is the line of credit in equilibrium. Note first that the retailer's profits from diverting are maximized by exhausting all credit, i.e., diverting yields βL . Thus, the retailer is indifferent when the profitability of producing $q_e \leq L$ equals βL . From the concavity of market profits, it follows that additional credit would lead the retailer to divert and thus cannot be profitable for the supplier. In contrast, smaller credit keeps repayment incentive compatible, but cannot increase supplier profits. More specifically, if the financial constraint is not binding ($q_e^u < L$), a marginally smaller line of credit has no effect in equilibrium. If, instead, it is optimal for the retailer to exhaust all credit ($q_e^u \geq L$), a smaller L must reduce supplier's profits whenever his sales are profitable, i.e., $\omega > c_u$. Hence, the supplier cannot do better than offering a credit line that makes the retailer indifferent between producing and diverting. This is, therefore, the incumbents' conjecture.

The credit policy (3.8) follows from solving $\pi_e = \beta L$ for L both when the financial constraint is binding and when it isn't. Figure 3.1 illustrates the two cases. If $\pi_e^u \geq \beta q_e^u$, the retailer makes higher profits by producing the optimal quantity of output q_e^u than by diverting the input required for such level of production and not honouring the debt. Then, the retailer is not financially constrained and it is indifferent between producing and diverting when the line of credit is L^u , which satisfies $\pi_e^u = \beta L$. Alternatively, when $\pi_e^u < \beta q_e^u$, if the retailer could borrow the optimal quantity of input for production, it would rather divert. Then, the supplier must limit the line of credit to make repayment incentive compatible. The retailer

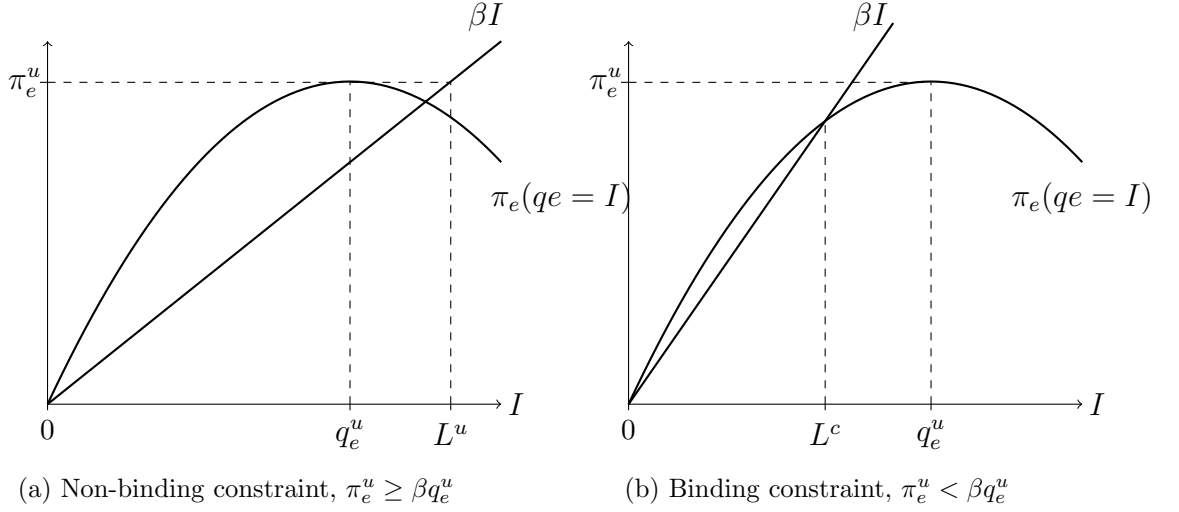


Figure 3.1: Line of credit extended by the supplier for a given input price. In (a) the constraint is not binding, so the supplier offers L^u units of input and the retailer borrows q_e^u . In (b) the constraint is binding, so the supplier offers L^c units and the retailer exhaust all credit. Parameter values are $M = 10$, $\omega = 2$, $c = 2$, $N = 5$ and $\beta = 0.5$ in (a), $\beta = 0.9$ in (b).

is indifferent when the credit line is L^c , which satisfies $\pi_e^c = \beta L$. Note in Figure 3.1 that $L^c = L^u$ when $\pi_e^u = \beta q_e^u$, so the line of credit is continuous on q_e^u . Thus, the retailer is financially constrained if and only if $\pi_e^u < \beta q_e^u$. As will become clear, this depends on the input price ω , which is strategically set by the supplier.

3.3.3 Input price

At $t = 0$ the supplier sets input price ω to maximize profits $\pi_u = (\omega - c_u)q_e$, subject to the credit policy L^* . Crucially, the price not only maximizes supplier's profits given a demand for input (production) in either equilibrium, i.e. for q_e^u or L^c , but it also determines the equilibrium itself. For instance, the price maximizing supplier's profits for a demand q_e^u must be such that the retailer is not financially constrained, i.e. $q_e^u \leq L^u$. The following proposition characterizes the supplier's pricing policy in equilibrium, which depends on market fundamentals.

Proposition 35 Define the price policies $\omega^u = \frac{M+(N+1)c_u+Nc}{2(N+1)}$ and $\omega^c = \frac{M+(N+1)(c_u-\beta)+Nc}{2(N+1)}$

satisfying $\omega^u > \omega^c$; and the policy $\bar{\omega} = \frac{M-(N+2)\beta+Nc}{N+1}$. Market fundamentals determine equilibrium outcomes such that

1. For $\bar{\omega} > \omega^u$ the supplier sets input price ω^u ; the retailer is not financially constrained and does not exhaust all credit;
2. For $\bar{\omega} \in [\omega^c, \omega^u]$ the supplier sets an input price $\bar{\omega} = \frac{M-(N+2)\beta+Nc}{N+1}$; the retailer is not financially constrained but exhausts all credit;
3. For $\bar{\omega} < \omega^c$ the supplier sets input price $\omega^c = \frac{M+(N+1)(c_u-\beta)+Nc}{2(N+1)}$; the retailer is financially constrained and therefore exhausts all credit.

I derive the price cutoffs of Proposition 35 in the Appendix; here I provide the main intuition. When the retailer is not constrained ($q_e = q_e^u$), the supplier maximizes profits by setting a price ω^u . Instead, if financial constraints are binding ($q_e = L^c$), the supplier's profits are maximized for ω^c . The relation $\omega^u > \omega^c$ shows that the retailer always pays a lower price for input when it is financially constrained. Notably, these pricing policies maximize supplier's profits *conditional* on the retailer's financial constraints. However, whether the retailer is constrained depends on the input price. The previous analysis showed that the retailer is constrained when $\pi_e^u < \beta q_e^u$. Here, price ω determines the profitability of producing π_e^u , and therefore whether there is credit rationing in equilibrium. A graphical argument follows from Figure 3.1. For a given input diversion value β , a small price ω makes producing relatively profitable, and the retailer is not constrained. Increasing ω lowers market profits, and financial constraints eventually become binding.

The pricing policy $\bar{\omega}$ satisfies $\pi_e^u = \beta q_e^u$, and is such that the retailer is financially constrained if and only if $\omega > \bar{\omega}$. When market fundamentals satisfy $\omega^u < \bar{\omega}$, there are no constraints in equilibrium. Intuitively, the supplier's price policy for a non-constrained retailer ω^u makes repayment incentive compatible for the optimal production q_e^u . Thus, unconstrained price and production are an equilibrium.

Moreover, the price policy for a constrained retailer ω^c does not provide incentives to divert, so financial constraints do not arise in equilibrium. Formally, it holds both $q_e^u(\omega^u) < L^u(\omega^u)$ and $q_e^u(\omega^c) < L^c(\omega^c)$. In this equilibrium, market outcomes are characterized by ω^u . Production levels are given by q_e^u and q_i^u in (3.4) and (3.5); and the line of credit is L^u in (3.8).

A similar intuition applies for $\omega^c > \bar{\omega}$, which implies that the retailer is financially constrained in equilibrium. In particular, ω^c is such that makes the retailer constrained, and thus it is an equilibrium. Furthermore, with a price ω^u repayment of debt corresponding to q_e^u is not incentive compatible, hence it is not an equilibrium price. More formally, we have $L^c(\omega^c) < q_e^u(\omega^c)$ and $L^u(\omega^u) < q_e^u(\omega^u)$. Here, equilibrium outcomes are characterized by input price ω^c and correspond to retailer's production L^c in (3.8) and incumbents' production q_i^c in (3.5).

When fundamentals are such that $\bar{\omega} \in [\omega^c, \omega^u]$, neither the unconstrained outcome nor the constrained one are an equilibrium for the associated price policies ω^u and ω^c . This is, with ω^u debt repayment is not incentive compatible when producing q_e^u ; with ω^c honouring the debt is incentive compatible for a production q_e^u . Then, the supplier sets price $\bar{\omega}$, which satisfies $q_e^u = L^u = L^c$. Thus, the retailer is not constrained, but it exhausts all credit. The result follows from the distinct responsiveness of optimal production and credit line to the input price. In particular, lowering the price would lead to a situation where $q_e^u < L^u < L^c$, so the retailer would not exhaust all credit. However, it would then be optimal for the supplier to set a higher price ω^u rather than a lower one. Similarly, a price higher than $\bar{\omega}$ yields $L^c < L^u < q_e^u$, thereby making the retailer financially constrained. But then the optimal price lower, i.e, ω^c , not higher.

3.3.4 Equilibrium outcomes

The following Corollary provides an intuitive overview of the main equilibrium outcomes that is convenient for subsequent discussion.

Corollary 36 *When the retailer does not exhaust all credit, i.e. for $\omega^u < \bar{\omega}$, there is double marginalization over the vertical chain, and market outcomes are those of a Cournot-Nash.*

With financial constraints, i.e. when $\omega^c > \bar{\omega}$, the retailer's production is that of a Stackelberg leader with a unit cost $c_u + \beta$, and incumbents act as laggard firms. Furthermore, supplier's profits are those of the Stackelberg leader, and retailer's profits equal its agency rents, i.e. $\pi_e^c = \beta L^c$.

The first statement is straightforward and highlights that contractual frictions need not affect market outcomes when agency costs are small. The second statement is proved formally in the Appendix; here I develop the main intuition. When the retailer is financially constrained ($\omega^c > \bar{\omega}$), it exhausts all credit. As a result, the input price determines the retailer's production ex ante and provides leadership to the retailer with respect to incumbents, which therefore become laggards. Moreover, because the retailer exhausts all credit, there is no double marginalization over the vertical chain. Hence, the supplier effectively delegates production to the retailer, conceding just enough rents to make repayment incentive compatible, i.e. β for each unit of input extended in credit. It then becomes optimal to set a price ω such that the line of credit equals to the production of a Stackelberg leader with a unit cost $c_u + \beta$. Here, c_u is the actual production cost whereas β is the retailer's agency rent per unit of input extended in credit.

3.4 Product Market Competition

I characterize equilibrium outcomes as a function of the number of incumbents and compare them with those of a game where there are no contractual frictions. Comparative statics shed light on the interaction between financial constraints and product market competition. The next Corollary follows directly from Proposition 35:

Corollary 37 Consider the cutoffs $N_1 = \frac{M-c_u-4\beta}{2\beta-c+c_u}$, $N_2 = \frac{M-c_u-3\beta}{\beta-c+c_u}$ and $\bar{N} = \frac{M-c_u-\beta}{\beta-c+c_u}$ that satisfy $N_1 < N_2 < \bar{N}$. Equilibrium is such that

1. If competitive pressure is low, i.e., $N < N_1$, the retailer is not financially constrained and does not exhaust all credit;
2. If competitive pressure is moderately low, i.e., $N \in [N_1, N_2]$, the retailer is not financially constrained but exhausts all credit;
3. If competitive pressure is moderately high, i.e., $N \in [N_2, \bar{N})$, the retailer is financially constrained and therefore exhausts all credit;
4. If competitive pressure is sufficiently high, i.e., $N \geq \bar{N}$, no credit is extended and the retailer does not enter the market.

Cutoffs N_1 and N_2 solve $\omega^u = \bar{\omega}$ and $\omega^c = \bar{\omega}$ respectively for the number of incumbents. Moreover, cutoff \bar{N} satisfies $\omega^c = c_u$. The results are illustrated in Figure 3.2, where I plot the main equilibrium outcomes as a function of N (solid). Moreover, I plot the outcomes of a setting where there are no contractual frictions (dashed). For this benchmark setting, I simply assume that input diversion yields zero revenues, so the retailer has no incentives to divert. Formally, if $\beta = 0$ it always holds that $\pi_e^u \geq \beta q_e^u$, and the line of credit is unlimited, i.e., $L^u \rightarrow \infty$.

The characterization of all functions plotted is given in the Appendix. With low competitive pressure ($N < N_1$) the retail market is relatively profitable and contractual frictions do not affect market outcomes. Corollary 36 establishes that the commercial relation between supplier and retailer is characterized by double marginalization, and production corresponds to a Cournot-Nash equilibrium. Figure 3.2 shows that input price ω^u decreases with competitive pressure N , but not enough to outweigh the reduced levels of production caused by higher competition, so q_e^u decreases too. More competition also raises agency costs, and therefore reduces the line of credit L^u . Crucially, L^u decreases at a higher rate than the optimal

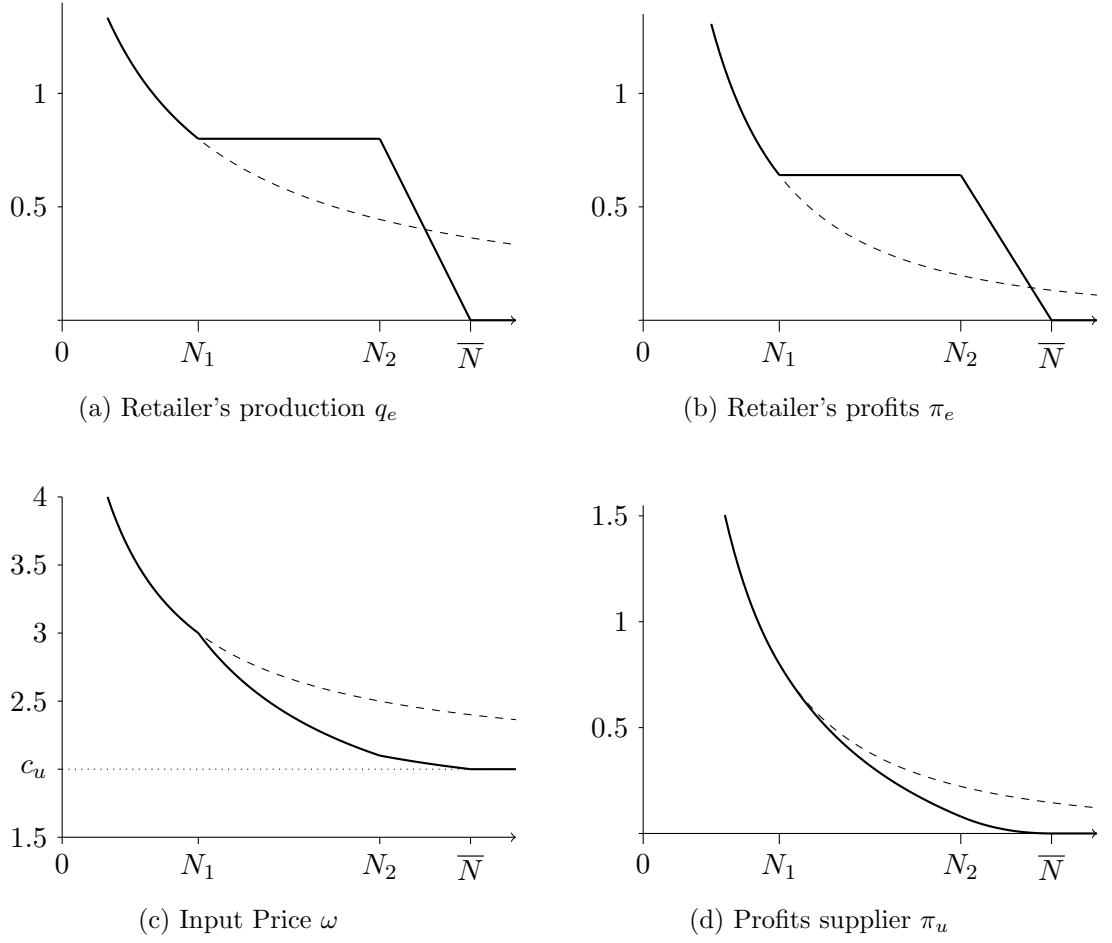


Figure 3.2: Equilibrium with contractual frictions (solid) versus equilibrium with no frictions (dashed). Parameter values for the setting with frictions are $M = 10$, $c_u = 2$, $c = 2$ and $\beta = 0.8$. The resulting cutoffs on the number of incumbents are $N_1 = 3$, $N_2 = 7$ and $\bar{N} = 9$. Parameter values for the setting with no frictions are equal except for $\beta = 0$.

production q_e^u and as a result, $q_e^u = L^u$ when $N = N_1$. For higher competitive pressure, double marginalization is no longer an equilibrium because $q_e^u < L^u$.

With a moderately small number of competitors $N \in [N_1, N_2]$ the supplier sets a price $\bar{\omega}$ so that the optimal production equals the credit line: $q_e^u = L^u = L^c$. Even though the retailer is not constrained, it acquires input at a lower price $\bar{\omega} < \omega^u$ and exhausts all credit. The benefits of this are twofold. First, a smaller marginal cost. Second, a first-mover advantage with respect to incumbents. Figure 3.2 shows

that the input price $\bar{\omega}$ is such that the retailer's optimal production q_e^u is invariant to the number of competitors and in turn, so are his profits. However, this is not an equilibrium when competition is strong enough ($N \geq N_2$). Then, the repayment of debt associated to this relatively large level of production $q_e^u(\bar{\omega})$ is no longer incentive compatible, and the retailer is financially constrained.

Moderately high competition $N \in [N_2, \bar{N})$ generates to financial constraints. The supplier sets a small input price than in a setting with no frictions ($\omega^c < \omega^u$), but the retailer cannot borrow enough to produce optimally, $L^c < q_e^u(\omega^c)$. Notably, Figure 3.2 shows that financial constraints can lead to higher production levels and higher profits for the retailer. From Corollary 36, it follows that this is because agency rents from producing the quantity of a Stackelberg leader are larger than the profits in a setting with double marginalization. Figure 3.2 shows that this holds when the number of incumbents is sufficiently close to N_2 , but not for higher competitive pressure.

As the number of incumbents grows, the production of a Stackelberg leader approaches zero due to the relative inefficiency with respect to incumbents: $c_u + \beta > c$. Notably, financial constraints provide a commitment device, but increase the supplier's cost of accessing the retail market. When $N \rightarrow \bar{N}$, market revenues converge to the effective marginal cost for the supplier, $c_u + \beta$. In particular, $\omega^c \rightarrow c_u$ to pay for the actual production cost, whereas the remaining β is captured by the retailer as an agency rent. With higher competitive pressure, i.e. $N \geq \bar{N}$, the supplier can no longer extend credit in a profitable way.

3.5 Optimal Contracts

In this section suppose that the supplier can set a non-linear input price. Notably, this is equivalent to considering the optimal contract for the supplier. The next proposition summarizes the main equilibrium outcomes:

Proposition 38 *In equilibrium, when the supplier can set non-linear prices, market outcomes are such that:*

- i) The supplier extends in credit a quantity of input L that equals the production of a Stackelberg leader with a unit cost $c_u + \beta$;*
- ii) The retailer exhausts all credit, i.e. $q_e = L$, and has profits that equal its agency rents, i.e., $\pi_e = \beta L$;*
- iii) The supplier's profits are those of a Stackelberg leader with a unit cost $c_u + \beta$.*

I argue that Proposition 38 follows directly from previous results. When the supplier can offer an optimal contract, retailer's profits from producing must be equal to its agency rents, i.e., $\pi_e = \beta L$. In particular, note that if retailer's profits were lower, it would have incentives to divert. Alternatively, if they were higher, the supplier could charge a bigger price while keeping repayment incentive compatible, and therefore it would not be maximizing profits. From this argument it also follows that the quantity of input extended in credit must equal retailer's production. Equivalently, in equilibrium the retailer must exhaust all credit. Otherwise, if $q_e < L$, the supplier would be granting the retailer with unnecessary rents to make repayment incentive compatible, and thus would not be maximizing profits. This demonstrates statement *ii)* of the proposition.

For statements *i)* and *iii)* consider the supplier's optimal production and profits. Since the retailer exhausts all credit, i.e. $q_e = L$, the credit line acts as a commitment device in the retail market. Thus, the retailer becomes a leading producer, and the supplier can both determine its production and extract all its surplus except for the agency rents. It then follows that the supplier maximizes profits by extending in credit the input that corresponds to the production of a Stackelberg leader with a unit cost $c_u + \beta$, where c_u is the actual production cost and β is the agency cost for each unit of input lent.

When the supplier can set a non-linear price, contractual frictions always benefit the retailer. In particular, retailer's profits equal its agency rents, and these are null when there exist no incentives to divert. Formally, $\pi_e = 0$ when $\beta = 0$. Notice also that retailer's profits depend on contractual frictions (input diversion value β), but not on the presence of financial constraints. This leads us to a relevant insight that is captured by the following Corollary:

Corollary 39 *With non-linear input prices, the retailer need not be financially constrained, despite contractual frictions and regardless of market fundamentals. The following two-part tariff satisfies this condition:*

$$\begin{aligned}\omega_{tpt} &= \frac{1}{2} \left[(N+2)(\beta + c_u) - \frac{N(M + Nc)}{(N+1)} \right], \\ F_{tpt} &= \frac{1}{4} [[M - N(\beta - c + c_u) - c_u - \beta] [M - N(\beta - c + c_u) - c_u - 3\beta]]\end{aligned}\tag{3.9} \tag{3.10}$$

where ω_{tpt} represents the unit price and F_{tpt} the fixed fee.

I derive $\{\omega_{tpt}, F_{tpt}\}$ in the Appendix. The unit price ω_{tpt} is such that in a setting with double marginalization, the retailer's production equals that of a Stackelberg leader with a unit cost $c_u + \beta$. The fixed fee F_{tpt} extracts the remaining part of retailer's surplus except for the agency rents, i.e. for βL . Notably, with this two-part tariff the retailer is never financially constrained because it can always produce optimally given the input price. In particular, provided that producing is profitable, optimal production is determined by the marginal cost ω_{tpt} . Here, the line of credit is such that the retailer can borrow just enough input to produce optimally.

Corollary 39 reveals a key limitation of the definition of financial constraints used here (Definition 31), which is seemingly standard. This is that financial constraints may not capture the presence of contractual frictions and agency costs, even though these affect market outcomes. The result highlights that taking production

costs as given when studying firms' credit rationing may neglect a crucial element of the whole picture, namely the fact that these costs are endogenous. The first part of the paper showed that with linear input prices financial constraints arise endogenously in equilibrium only if agency costs are sufficiently large. In this section I show that with non-linear prices the retailer might never be financially constrained, even though market outcomes are affected by financial frictions.

3.6 Concluding Remarks

In this paper I model an endogenous relation between financial constraints and product market competition. I characterize a trade credit transaction where a supplier lends input to a competitive retailer, and contractual frictions may lead to financial constraints. By setting the input price, the supplier determines both the retailer's optimal demand for input and the quantity of input that can be extended in credit while making repayment incentive compatible. Credit rationing arises in equilibrium when the retailer can not borrow enough to produce optimally for a given input price.

When the supplier sets linear prices, the retailer is financially constrained if competitive pressure is strong enough. I show that a financially constrained retailer faces lower input prices, and it can make higher profits due to its own financial constraints. Formally, this occurs when its agency rents are larger than the profits of double marginalization in a setting with no frictions. When non-linear input prices are considered, the retailer might never be financially constrained despite the presence of contractual frictions and regardless of market fundamentals. The result reveals that seemingly standard definitions of financial constraints may neglect a key part of the picture explaining financial agreements, namely the endogeneity of production costs.

3.7 Appendix

3.7.1 Proof of Lemmas 33-34

Lemma 33. Incumbents observe ω and have the right conjecture of L , so they know whether $q_e^u(q_i) \leq L$ or $q_e^u(q_i) > L$. When $q_e^u(q_i) \leq L$, equilibrium production levels $\{q_e^u, q_i^u\}$ solve the system of best response functions (3.3) and (3.2), which are derived in the main text. Note that when $q_e^u(q_i) > L$, the retailer exhausts all credit because market profits are quasiconcave in q_e with a maximum at q_e^u . Incumbents' response then is $q_i(L) = \frac{M-L-c}{N+2}$.

Profits functions are obtained by plugging in the production levels derived above so that $\pi_e^* = (M - Nq_i^* - q_e^* - \omega)q_e^*$ and $\pi_i^* = (M - Nq_i^* - q_e^* - c)q_i^*$ for retailers.

Lemma 34. The expression for L^u follows from solving $\pi_e^u = \beta L$ for L . In particular, the equation reads

$$\left[\frac{M - (N+1)\omega + Nc}{N+2} \right]^2 = \beta L^u. \quad (3.11)$$

To obtain L^c I solve $\pi_e^c = \beta L$ for L . This is,

$$L^c \left[\frac{M - (N+1)\omega - L^c + Nc}{N+1} \right] = \beta L^c. \quad (3.12)$$

Incumbents' conjecture is correct because the supplier has no incentives to deviate and offer a different line of credit.

3.7.2 Proof of Proposition 35 and Corollary 36

Proposition 35. Lemma 32 shows that $I = q_e$, thus supplier's profits read $\pi_u = (\omega - c_u)q_e$. Suppose that the retailer is not financially constrained, so he borrows

q_e^u . Then the supplier sets input price to maximize

$$\pi_u = (\omega - c_u) \left[\frac{M - (N + 1)\omega + Nc}{N + 2} \right]. \quad (3.13)$$

The solution is given by ω^u in Proposition 35. Similarly, suppose that the retailer is constrained and thus he borrows L^c . Then the supplier sets a price to maximize

$$\pi_u = (\omega - c_u) [M - (N + 1)(\omega + \beta) + Nc]. \quad (3.14)$$

The solution is given by ω^c in Proposition 35.

For prices ω^u and ω^c to be an equilibrium they must be such that the retailer is not constrained and constrained respectively. The retailer is constrained when $\pi_e^u < \beta q_e^u$. There exists a unique solution $\bar{\omega}$ for $\pi_e^u = \beta q_e^u > 0$ that follows from solving

$$\left[\frac{M - (N + 1)\omega + Nc}{N + 2} \right]^2 = \left[\frac{M - (N + 1)\omega + Nc}{N + 2} \right] \beta. \quad (3.15)$$

Thus, the retailer is financially constrained if and only if $\omega > \bar{\omega}$. Note that the same price function is obtained by solving $q_e^u = L^c$, indicating that this price equals the production in the constrained and the unconstrained equilibria.

Note that $\omega^u > \omega^c$. When $\bar{\omega} > \omega^u > \omega^c$, the retailer is not financially constrained neither for a price ω^u nor for ω^c . Thus, in equilibrium there are no financial constraints and the supplier sets a price ω^u . When $\omega^u > \omega^c > \bar{\omega}$ the retailer is financially constrained both with prices ω^u and ω^c . Thus, in equilibrium the retailer is constrained and the supplier sets a price ω^c .

When $\bar{\omega} \in [\omega^c, \omega^u]$ neither of the above is an equilibrium. Note that $\bar{\omega}$ satisfies $q_e^u = L^u = L^c > 0$, i.e.

$$\frac{M - (N + 1)\bar{\omega} + Nc}{N + 2} = \frac{1}{\beta} \left[\frac{M - (N + 1)\bar{\omega} + Nc}{N + 2} \right]^2 = M - (N + 1)(\bar{\omega} + \beta) + Nc > 0. \quad (3.16)$$

Furthermore, we have $q_e^u < L^u < L^c$ for $\omega < \bar{\omega}$ and $q_e^u > L^u > L^c$ for $\omega > \bar{\omega}$. Thus, neither a price $\omega < \bar{\omega}$ or a price $\omega > \bar{\omega}$ can be an equilibrium. With a price $\omega = \bar{\omega}$ the supplier maximizes profits given the input demand and the line of credit and therefore it is an equilibrium.

Corollary 36. When the financial constraint is binding ($\bar{\omega} < \omega^c$) the retailer's production is $L^c(\omega^c)$ or, equivalently,

$$L^c = \frac{M - (N + 1)(c_u + \beta) + Nc}{2}. \quad (3.17)$$

This also corresponds to the supplier's input sales. Supplier's profits are $\pi_u^c = (\omega^c - c_u)L^c$ whereas retailer's profits are given by $\pi_e^c = (M - Nq_i^c - L^c - \omega^c)L^c$. Plugging both ω^c and L^c into the profit functions yields

$$\pi_u^c = \frac{[M - N(\beta - c + c_u) - c_u - \beta]^2}{4(N + 1)} \text{ and } \pi_e^c = \beta L^c \quad (3.18)$$

Denote q_L the production of a Stackelberg leader with unit cost $c_u + \beta$. From (3.2) it follows that incumbents' best reply reads $q_i(q_L) = \frac{M - q_L - c}{N + 1}$. Thus, the leader maximizes profits $\pi_L = (M - Nq_i(q_L) - q_L)q_L - (c_u + \beta)q_L$. The first order condition yields

$$q_L^* = \frac{M - (N + 1)(c_u + \beta) + Nc}{2}. \quad (3.19)$$

Plugging this expression into the profit function I obtain

$$\pi_L^* = \frac{[M - N(\beta - c + c_u) - c_u - \beta]^2}{4(N + 1)} = \pi_u^c \quad (3.20)$$

3.7.3 Proof of Corollary 37 and characterization of Figure 3.2

Corollary 37. Consider the expressions for ω^u , ω^c and $\bar{\omega}$ in Proposition 4. Solving $\omega^u = \bar{\omega}$ for N yields the cutoff N_1 , which satisfies $\bar{\omega} > \omega^u$ if and only if $N < N_1$. Similarly, solving $\omega^c = \bar{\omega}$ for N yields the cutoff N_2 , which satisfies $\omega^c > \bar{\omega}$ if and

only if $N > N_2$.

The cutoff \bar{N} is obtained by solving for the input price that equals the marginal cost under financial constraints, i.e. solving $\omega^c = c_u$ for N . The cutoff also satisfies a null line of credit, so it can be derived by solving $L^c = 0$ for N .

Figure 3.2. The full characterization of cutoffs N_1, N_2 and \bar{N} , and input prices ω_u , $\bar{\omega}$ and ω_c , is given by Proposition 4.

When $N < N_1$, production levels and profits are obtained by plugging ω_u into $\{q_e^u, q_i^u, \pi_e^u, \pi_i^u\}$ in Lemma 2, whereas supplier's profits satisfy $\pi_u^u = (\omega_u - c_u)q_e^u$. The exercise yields

$$\begin{aligned} q_e^u &= \frac{M - (N+1)c_u + Nc}{2(N+2)} & \pi_e^u &= (q_e^u)^2 \\ q_i^u &= \frac{M - 2c + \frac{M + (N+1)c_u + Nc}{2(N+2)}}{N+2} & \pi_i^u &= (q_i^u)^2 \\ \pi_u^u &= \frac{[M - (N+1)c_u + Nc]^2}{4(N+1)(N+2)} \end{aligned}$$

When $N \in [N_1, N_2]$, production levels and profits are obtained by plugging $\bar{\omega}$ into the same functions. I obtain

$$\begin{aligned} q_e^u &= \beta & \pi_e^u &= (q_e^u)^2 \\ q_i^u &= \frac{M - c - \beta}{N+1} & \pi_i^u &= (q_i^u)^2 \\ \pi_u^u &= \beta \left[\frac{M - \beta(N+2) + Nc}{N+1} - c_u \right] \end{aligned}$$

When $N \in (N_2, \bar{N}]$, production levels and profits are obtained by plugging ω_c into L^c in Lemma 33 for the retailer's production; both ω_c and $L^c(\omega_c)$ and into $\{q_i^c, \pi_e^c, \pi_i^c\}$ and into $\pi_u^c = (\omega_c - c_u)L^c$ for the remaining functions. It yields

$$\begin{aligned} L^c &= \frac{M - (N+1)(c_u + \beta) + Nc}{2} & \pi_e^c &= \beta L^c \\ q_i^c &= \frac{M + N(\beta - c + c_u) - 2c + c_u + \beta}{2(N+1)} & \pi_i^c &= \left(\frac{M - c}{2(N+1)} \right)^2 - \frac{(\beta - c + c_u)^2}{4} \\ \pi_u^c &= \frac{[M - N(\beta - c + c_u) - c_u - \beta]^2}{4(N+1)} \end{aligned}$$

3.7.4 Proof of Corollary 39

The supplier can set an optimal contract where the retailer is not financially constrained by offering input at a price such that optimal retailer's production is q_L^* and extracting his surplus with a fixed fee so that $\pi_e = \beta q_L^*$.

The equilibrium production levels in a setting with no financial constraints q_e^u and q_i^u are provided by (3.4) and (3.5) in the main text. Solving $q_e^u = q_L^*$ for ω yields ω_{tpt} in the corollary. This unit price generates the input demand that equals the optimal production of a Stackelberg leader with unit cost $\beta + c_u$.

For the supplier to extract all the retailer's surplus while making repayment incentive compatible he must set a fixed fee $F = (M - Nq_i^u - q_L^*)q_L^* - (\omega + \beta)q_L^*$. The fixed fee equals retailer's market revenues net of the unit price ωq_L^* and the agency cost βq_L^* . Algebra manipulation yields F_{tpt} . Solving $F = 0$ for N shows that the fixed fee is negative when $N \in (N_2, \overline{N})$.

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